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A new version of the HERMES model

HERMES III

November 2013

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Abstract – This Working Paper is aimed at describing the current version of Federal Planning Bureau's medium-term macrosectoral model, named HERMES. This model is used to produce on a regular basis medium-term outlooks for the Belgian economy. In addition to the main macroeconomic aggregates (GDP, private consumption, external trade, investments,…), those outlooks also concern labour market aggregates, detailed public finances, energy consumption and greenhouse gas emissions. The HERMES model is also used to compute the impact of policy measures and external shocks on the Belgian economy.

Jel Classification – C5, E1, E2, E6, H2, H5, J3

Keywords – HERMES, econometric model, medium-term, macroeconomic outlooks

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Executive summary

This Working Paper presents the main features of the HERMES III model of the Belgian economy. HERMES[1] III is the third official version of the medium-term macroeconometric model developed by the Federal Planning Bureau (FPB). The model is suitable for medium-term outlooks and for the analysis of the effects of both economic policy and exogenous shocks on the Belgian economy.

HERMES became operational in the mid-eighties. The first version[2] was built during the period 1982-1986. The second version[3], which included several adjustments, notably with respect to environment and in relation with the structure of industries, was developed during the period 1995-2000. Since then the model has been maintained and developed on a regular basis. The most important developments of the model in the recent years have been:

- the disaggregation of employment of each industry by wage category (low-pay jobs, high-pay jobs and special employment programmes) and by age category (workers aged less than fifty and workers aged fifty or more);
- the further disaggregation of public finances: the former single account for all communities and regions has now been replaced by four separate accounts (Brussels Region, Walloon Region, Flemish Region and Flemish Community, and French Community);
- the implementation of a new health care module;
- the new European nomenclature of activities (NACE REV2) and its transposition at the Belgian level (NACE-BEL 2008), operational since 2011, have been taken into account, which implied redefining HERMES' industries;
- besides CO_2, other greenhouse gases, such as methane CH_4, nitrous oxide N_2O and the fluorinated gases (HFK's, PFK's and SF_6) which are related to the activity of specific branches, were also introduced.

Moreover the main blocks of the original model, such as the production block, the private consumption block, external trade, … were completely reestimated last year and during the first half of 2013 (mainly on the period 1980-2011, i.e. on a sample of about 32 observations) and sometimes also (partially) revised.

This Working Paper is not meant to provide a comprehensive encyclopedic description of HERMES III. The purpose is rather to survey its main blocks. The text is organized as follows: chapter 1 describes the general characteristics of the model. Chapters 2 to 6 deal with the main blocks of the model. We focus on households' demand (private consumption and housing investments - Chapter 2), external trade (Chapter 3), production (Chapter 4), energy and greenhouse gas emissions (Chapter 5) and public finances (Chapter 6). Finally (Chapter 7), the two main uses of the model are described. Next to the elaboration of medium-term outlooks (which implies the computation of potential growth and output

1 Harmonized Econometric Research for Modelling Economic Systems.
2 For a presentation of the first version of HERMES, see Bossier et al. (1989).
3 For a presentation of the second version of HERMES, see Bossier et al. (2000).

gap), HERMES is also used to make variants, that is to compute the impact on the Belgian economy of economic policy measures as well as external shocks. In order to illustrate the main properties and mechanisms of the model, the results of four variants are presented: an increase in public investment, a reduction in employers' social security contributions targeted at low-wage workers, a higher VAT on private consumption and an oil price increase.

1. General presentation of the model

This document is intended to present the latest version of the macrosectoral HERMES model completed by September 2013. HERMES (Harmonized Econometric Research for Modelling Economic Systems) is a model of the Belgian economy which breaks down the activity among 15 productive industries (see below for the decomposition of the activity). This model was developed at the Federal Planning Bureau and is used by the latter for the following applications: short-term and medium-term projections, assessment of the effects of economic policy measures, production of variants.

General characteristics

The HERMES model belongs to the large structural econometric models, since it has a total of about 8 000 equations (but a large portion are accounting or definition equations). Out of this total, about 600 equations are estimated econometrically (behavioural equations). The relatively impressive size of the model is explained by the degree of disaggregation of activity (number of distinguished activities), but also by the fact that many of the blocks in the model have been extensively developed (detailed allocation of private consumption; comprehensive public finances; segmented labour market; energy-environment block distinguishing different greenhouse gases and 8 energy products, etc.) The modeller has thus the ambition to provide a sufficient detail of the Belgian economic activity (while not going to a detail that might suggest the availability of input-output to more than 60 industries) and to develop an energy block giving a sufficiently accurate picture of the energy consumption per agent represented in the model (and, in parallel, their greenhouse gas emissions). The choice was also to develop a public finances block able to provide complete and detailed accounts for the federal level as well as for the other levels of the public administrations.

Table 1: General characteristics of HERMES

- 8 000 equations (of which about 600 behavioural equations econometrically estimated)
- 1 200 exogenous variables (world trade, external prices, external interest rates, demography and labour supply, fiscal policy, etc.)
- 2 600 scalars (econometrically estimated parameters)
- 15 industries; 15 main consumption categories (total of 22); 6 main energy types (total of 13); 5 greenhouse gases
- 6 main institutional sectors (total of 12)
- Public administrations divided into 4 main entities (for a total of 7)
- Estimation period: 1980y1-2011y1 (sample of 32 yearly observations)

Industries	**Consumption categories**
Agriculture	Food, beverages and tobacco
Energy	- Food
Intermediate goods	- Non alcoholic beverages
Equipment goods	- Alcoholic beverages
Consumer goods	- Tobacco
Construction	Clothing and footwear
Transport and communication	Gross rent
- Rail and road transportation	Fuel for heating
- Air, maritime and fluvial transportation	- Solid fuels
- Other transport services and communication	- Liquid fuels
Trade, lodging and catering services	- Natural gas
Services of credit and insurance institutions	Power
Health care	Domestic services
Other market services to households and firms	Furniture and household equipment
Public administrations and education	Personal transport equipment
Other non-market services	Fuels for personal transport
	- Motor spirit
	- Diesel
	- Others
	Purchased transport
	- Rail transportation
	- Road transportation
	- Other transport services
	Communication services
	Medical care and health service
	Recreation, education, culture
	Other goods and services
	Tourism abroad

Energy and environment	Institutional sectors
Coal	Households
Coke	Non-profit institutions serving households (NPISH)
Crude oil	Corporate enterprises
Petroleum products	General Government
- Gas/diesel oil	- Federal government
- Residual fuel oil	- Regions and Communities
- Motor spirit	Brussels Region
- Diesel for transportation	Walloon Region
- Jet fuels	Flemish Region and Flemish Community
Gasses	French Community
- Natural gas	- Local authorities
- Coke oven gas	- Social security fund
- Blast-furnace gas	Rest of the world
Electricity	
Other fuels	
Renewables (biomass, wind energy, hydro energy, solar heat, etc.)	
Greenhouse gasses	
- CO_2 from fuel combustion	
- CO_2 from industrial processes and waste incineration	
- CH_4	
- N_2O	
- HFC, PFC, SF_6	

The HERMES model, as most macroeconomic models used for forecasting and economic policy studies, is part of what is often referred to as the "neo-keynesian synthesis", offering a synthesis between Keynesian and neo-classical ideas.

This model considers that the output (activity) is determined in the short term by demand: the short-term equilibrium is achieved through the traditional mechanisms such as the accelerator in the investment function and the income multiplier. This model also retains the hypothesis of short-term price rigidity and the persistence of imbalances on the labour and capital markets (imbalances which should theoretically disappear in the long term). On this short-term horizon, the model can be used to study the influence of fiscal and monetary policies on the demand components. Medium-term supply mechanisms gains will gradually take over and bring back, if necessary, the economy on a balanced growth path. The supply potential of the economy is introduced in the model via a supply block (production functions) which integrates structural factors such as demographics, technical progress or productivity gains. The medium-run adjustment will depend in practice on the functioning of the wage-price loop and the interaction of this loop with the real block of the economy. In particular, if demand is high and boosts employment, there will be a decline in unemployment, which will gradually lead to wage pressures and therefore feed price increases that will damage competitiveness. As a result, exports will decrease, as well as aggregate demand, which will ensure that the economy will come back to its long-run growth path.

Another equilibrating mechanism could refer to monetary and financial conditions. However, it should be noted that the HERMES model, unlike other macroeconomic models, doesn't incorporate a mechanism endogenising interest rates. Indeed, Belgian rates depend on European rates, themselves exogenously introduced in the model. It remains that changes in financial conditions will affect the actual variables in the model in a direct way: this interaction will also help to stabilise the long term-trajectory of the model.

Main blocks

- The real block: this block is the center of the model; it studies the evolution of the main real variables and how they interact with each other. This real block can be itself divided into more or less homogeneous sub-blocks. In the case of the HERMES model, one can distinguish:
  - The household spending: by far the largest component of aggregate demand, which usually includes the household consumption and residential investment. In HERMES, the consumption block has two levels: in a first level, there is a function of overall consumption; the second level contains a series of relations allocating total household consumption among the different categories of consumption expenditure.
  - The production block which determines the production factors' demand of each industry (labour, capital, energy, other intermediary inputs). It should be noted that with respect to labour, a distinction is made between different categories of workers (employment low and high wages, employment of "older" workers, special jobs). The demand for energy is also subject to a segmentation according to the product consumed by the firm (fuel types). As far as the capital factor is concerned, we note that gross fixed capital formation is subdivided into 6 categories[4]. Finally, intermediate demand is broken down into the deliveries of the various branches of production using an input-output table.
  - The external trade block: volumes and prices of exports and imports of goods and services.
- The wage-price block which specifies the formation of costs and prices. Special attention is paid to the formation of wages, which is studied, in coherence with the modelling of jobs (see above), based on the different categories of employees distinguished by the model (high wages, low wages, wages of older workers, special programmes, etc.).
- The income block that calculates for the institutional agents their disposable income and their financing capacity. It is in this block, which is mainly composed of accounting equations, that are computed the main global balances (capacity or borrowing of the country).
- The public finance block: can be considered as part of the income block. This block is highly developed in the HERMES model, since it contains not less than 2 200 equations, for most accounting equations, but also for a large number of "quasi-definition" equations.
- The financial and monetary block: it includes the computation of short-term as well as medium to long-term interest rates: the three-month Treasury bill rate; the yield on government bonds with

[4] Cultivated assets; Equipment except transport equipment (that is office machinery and hardware; radio, tv and communication equipment; other machinery and equipment); Transport equipment; Dwellings; Other buildings and structures; Computer software and other intangible fixed assets.

contractual maturity of over 6 years and the mortgage rate. This block also contains exchange rates and asset prices.

– The energy-environment block: specific to the HERMES model, the energy is both considered as a branch of activity generating a value added, operating surplus, investment, jobs,..., but also as a factor used by the different institutional agents (households, companies, public administrations,...). The highlight of the consumption of energy by product allows deriving CO_2 emissions from fuel combustion, but the model also calculates the other greenhouse gases.

The following chapters describe the main model blocks.

Data

Concerning the historical data taken into account in HERMES, national accounts[5] are the main source. They are currently defined according to ESA95.

The classification of economic activities currently used at European level is called NACE REV2. The transposition of this nomenclature of activities at Belgian level is called NACE-BEL 2008. As already mentioned, HERMES distinguishes 15 industries, the definition of which is presented in table 2.

Note that the HERMES industries are generally defined from the A38 level of disaggregation of NACE-BEL 2008. One exception however: it concerns the sub-branches of the branch "Transport and communication" whose definition requires to use the A64 level of disaggregation of NACE-BEL 2008.

The latest Input-Output tables, which are relative to the year 2005, are also taken into account in the model (as technical coefficients)[6].

Table 2: HERMES' industries definition (according to NACE-BEL 2008)

HERMES' industries	Definition according to NACE-BEL 2008
Agriculture (A)	AA
Energy (E)	CD+DD+EE
Intermediate goods (Q)	BB+CE+CF+CG+CH
Equipment goods (K)	CI+CJ+CK+CL
Consumer goods (C)	CA+CB+CC+CM
Construction (B)	FF
Transport and communication (Z)	HH+JB
. Rail and road transportation (Z_12)	49 (A64)
. Air, maritime and fluvial transportation (Z_34)	50+51 (A64)
. Other transport services and communication (Z_56)	52+53+61 (A64)
Trade, lodging and catering services (HA)	GG+II
Services of credit and insurance institutions (CR)	KK
Health care (SA)	QA+QB
Other market services to households and firms (OS)	JA+JC+LL+MA+MB+MC+NN+RR+SS
Public administrations and education (LM)	OO+PP
Other non-market services (DOM)	TT

5 Belgian national accounts are available on the website www.belgostat.be.

6 For a description of the 2005 Input-Output tables, see Avonds et al. (2010).

2. Demand of households: consumption and investment

2.1. Macroeconomic consumption

Private consumption is first determined at its macroeconomic level (this point)[7]. The obtained global amount is then allocated to the different consumption categories (see point 2.2 *infra*).

Consumer behaviour in the model is based on the life cycle hypothesis of consumption. According to this hypothesis, rational consumers attempt to maximise their intertemporal utility and find it optimal to smooth their consumption over time. Their consumption decision is based on a discounted stream of current and future expected net income (PI) and on their current stock of financial wealth (FW). Private consumption (C) can be represented as follows:

$$C = \rho(PI + FW) \tag{1}$$

If, as assumed by the life cycle hypothesis, all consumers were forward looking with perfectly functioning financial markets, they would be able to smooth their lifetime consumption. In reality a substantial proportion of consumers does not behave in this way for a variety of reasons, e.g. market imperfections, uncertainty, myopia and backward-looking behaviour. Thus, over the long run, consumption is a positive function of lifetime (human and non-human) wealth while in the short run it appears also to be constrained by current disposable income. The interest rate – as a proxy variable of the discount rate in the life cycle theory – should also play a role in explaining private consumption, although two opposite effects (an income effect and a substitution effect) interplay *a priori*. Age structure effects in the long run were also tested since the life cycle hypothesis predicts that age influences individuals' saving and consumption behaviour[8].

Financial wealth is approximated by the financial assets (denoted AH) held by households[9]. Concerning human wealth, it is standard in the literature to approximate it by current labour income. However, as pointed out by Al-Eyd et al. (2006, p. 8), wealth (including housing wealth) may be held for its income flow, among many possible motives. This is why we use total real personal disposable income (which includes income flows from wealth), or alternatively, the different income components (labour income, financial income, etc.). Our age structure variable is calculated as the ratio of 'prime-savers-aged' persons to the rest of the adult population (Erlandsen and Nymoen (2008)):

$$AGE = \frac{Population\ 30 - 39\ years\ old}{Population\ 20 - 29\ years\ old\ and\ 40 + years\ old} \tag{2}$$

7 For an overview of the literature over aggregate private consumption, see e.g. Bayar and Mc Morrow (1999). Different aspects of private consumption in Belgium have also been analysed in Bossier et al. (1995), Eugène et al. (2003) and Al-Eyd et al. (2006).

8 As stressed by Al-Eyd et al. (2006, p. 10), it is known since the pioneering work by Fair and Dominguez (1991) that the "omission of demographics is likely to lead to errors in analysing and forecasting consumption".

9 The financial assets held by households include time and savings deposits, foreign assets, term deposits, savings certificates and insurance certificates subject to previous deduction, collective investment undertaking, bonds and tax-exempt insurance certificates.

Indeed, in Belgium, the age group 30- to 39-year-old persons would have the smallest average propensity to consume (Ledent, 2009). Given the post-war demographics in Belgium, especially the fall of the fertility rate from 1965 onwards, this ratio raised from 0.22 in 1980 to 0.26 in 1995 before declining progressively to reach a level of 0.21 in 2011.

We used the two-step Engle-Granger estimation procedure: first, we estimated the long-term equilibrium relationship between consumption, income, wealth, the interest rate and the age structure variable; in a second stage, we incorporated the estimated long-run relation in the dynamic short-term specification as an error correction model (ECM).

In the first step, imposing homogeneity across income and wealth effects, the following long run equation was estimated:

$$\ln C_t = \alpha + \beta \ \ln\left(\frac{YDH}{PCH}\right)_t + (1-\beta)\ln AH_t + \gamma \ RRS_t + \delta \ AGE_t + E_t \tag{3}$$

where household consumption expenditure (C) and financial wealth (AH) are expressed in constant prices per capita; YDH is the per capita disposable income of households; PCH is the consumer price index and RRS the short-term real interest rate. We expect β to be positive and, on the basis of the life cycle theory, a negative sign for the age structure variable ($\delta < 0$) and the interest rate ($\gamma < 0$) should also be expected. Indeed, the income effect of changes in the interest rate is already captured by the income variable so that our econometric approach should reveal the negative intertemporal substitution effect on consumption.

The results of the estimation of equation (3) are given in table 3. This table also details the results for two other regressions, the first one omitting the age structure variable and the second one scrapping both the latter variable and the interest rate. For each regression, the standard ADF test on the residuals (cointegration test) is given in the two last columns.

We first notice that a relation incorporating only consumption, financial wealth and income would not be cointegrated (as shown by the ADF test on E_1). Moreover, the results indicate that consumption, financial wealth, the interest rate and disposable income would be cointegrated at the 5% significance level (E_2)[10]. More importantly, cointegration seems even stronger (significant at the 1% level), and the long-run estimated wealth elasticity of consumption is somewhat higher (0.05), whenever one incorporates the age structure variable into the relationship (E_3). According to our results, a one percentage point increase in AGE would decrease consumption per capita by about 0.5% in the long run. In other words, the recent evolution of the ‘prime-savers’ ratio would have stimulated Belgian aggregate consumption, other things equal. Consumption would similarly decrease by about 0.5% in the long run when the real interest rate increases with one percentage point, ceteris paribus[11].

[10] This result is very close to the one obtained by Eugène et al. (2003) who found evidence of cointegration between the three first variables and human wealth – calculated from data on labour income – rather than total disposable income.

[11] This result is close to the one obtained by Eugène et al. (2003), i.e., 0.57%.

Table 3: Long run cointegrating relations ADF tests (sample: 1980-2011)

	β	γ	δ	ADF Test	ADF 5%
$\ln C = \alpha + \beta \ln\left(\frac{YDH}{PCH}\right) + (1-\beta)\ln AH + E_1$	0.91			-1.87	-2.96
$\ln C = \alpha + \beta \ln\left(\frac{YDH}{PCH}\right) + (1-\beta)\ln AH + \gamma\, RRS + E_2$	0.98	-0.76		-3.87	-2.96
$\ln C = \alpha + \beta \ln\left(\frac{YDH}{PCH}\right) + (1-\beta)\ln AH + \gamma\, RRS + \delta\, AGE + E_3$	0.95	-0.49	-0.48	-4.31	-2.96

Turning now to the short term, we split income into two components: the net property income (IDH), on the one hand, and $YDH - IDH$, on the other hand, the latter component being a proxy for labour and transfer incomes. As already mentioned, the dynamics of private consumption is specified as an ECM in HERMES:

$$\Delta \ln C_t = \alpha_0 + \gamma_1 \Delta \ln\left(\frac{YDH - IDH}{PCH}\right)_t + \gamma_2 \Delta \ln\left(\frac{IDH}{PCH}\right)_t + \gamma_3 \Delta \ln AH_t + \gamma_4 \Delta\, RRS_t + \gamma_5 \Delta\, UR_t + \lambda\, E_{3,t-1} \tag{4}$$

where

UR is the unemployment rate (definition FPB), as an indicator for the confidence degree of consumers;

$E_{3,t-1}$ is the deviation of consumption, in year t-1, from the long-run relation (3).

The parameter λ must be negative (error correction) and measures the speed of adjustment to the long-term equilibrium. The results from estimation (on the period 1981-2011) of equation (4) are as follows:

Table 4: Coefficients of the macroeconomic consumption function

Coefficient	λ	γ_1	γ_2	γ_3	γ_4	γ_5
Estimation value	-0.27	0.31	0.11	0.04	-0.36	-0.36
t-test	-2.12	3.58	4.79	2.13	-3.01	-1.68
Other tests	Adjusted R^2	F-Stat	Durbin-Watson	Log-likelihood		
	0.58	6.12	2.12	114.39		

The estimated value of λ is consistently negative: any deviation of consumption from its long-term path would be corrected after four years. As shown in table 4, we found different elasticities of consumption for the different components of income in the short run: the elasticity on short term labour and transfer income is fairly high whereas the sensitiveness of consumption to property income is lower. This equation also confirms the very limited role for the financial assets. These results (weak wealth effect, stronger labour income effect) suggest significant liquidity constrained behaviour in Belgium, i.e., little ability for households to finance their current consumption out of wealth. Still, the significant difference term for the real interest rate would confirm the effect on consumption of intertemporal substitution. Also note a relatively high coefficient in absolute value for the unemployment rate.

Elasticities of private consumption

The following table gives the elasticities of the private consumption w.r.t., respectively, labour and transfer income and real net property income. The sensitiveness of private consumption to unemployment is also displayed.

Table 5: Private consumption elasticities

	t	t+3	t+5	Long term
Real labour and transfer income	0.31	0.61	0.70	0.82
Real net property income	0.11	0.12	0.12	0.13
Unemployment	-0.36	-0.14	-0.08	0.00

As can be seen from table 5 a permanent increase of 1% of the real labour and transfer income increases consumption with 0.61% after 4 years and about 0.8% in the long run. The estimated elasticity of consumption on property income (*IDH*) is about 0.1 both in the short and the long run. It is lower than the elasticity on labour and transfer income due to the lower weight of *IDH* in total disposable income. Note also that an increase of 1% of unemployment rate would depress consumption by 0.4% (and thus would increase the savings rate) in the short term. This negative effect disappears in the long term.

2.2. Consumption allocation module[12]

In this section we specify an allocation model of private expenditures, embedded in a generalisation of Deaton & Muellbauer's (1980) Almost Ideal Demand System (AIDS). The main advantage of specifying a complete demand system is that this is the only way to take into account the fact that all household consumption decisions are subject to the same budget restriction. By implication, every determinant of spending on any consumption is a potential determinant of spending on all or some other goods and services.

The section is organized as follows. Point 2.2.1 presents the specification of the model. Taking the original AIDS specification as a starting point, the model is extended by introducing a dynamic adjustment mechanism and by the inclusion of demographic variables. These were expected to capture shifts in consumption patterns related to the changing age composition of the population. Point 2.2.2 provides a brief overview of the data used to estimate the model. The estimation results, more precisely the implied income and price elasticities, are discussed in point 2.2.3.

2.2.1. Specification of the consumption allocation module

a. The static AIDS model

One of the most widely used consumption allocation models in empirical research is the Almost Ideal Demand System (AIDS) (Deaton & Muellbauer, 1980). Its popularity stems from its generality (combining appealing features of both the Rotterdam and the translog models) and the ease with which (restrictions on) its parameters can be estimated. The demand system is specified in terms of a set of

12 See, for instance, Bracke, I. and Willemé, P. (2012).

budget share equations of the following form (all variables measured in current period, observation subscripts dropped for clarity):

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \beta_i \ln\left(\frac{X}{P}\right), \qquad i = 1, \ldots, n; j = 1, \ldots, n \tag{5}$$

Where

$$\ln P = \alpha_0 + \sum_i \alpha_i \ln P_i + 0.5 \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j \qquad i = 1, \ldots, n; j = 1, \ldots, n \tag{6}$$

w_i = the i-th budget share

P_i = the price of the i-th commodity

X = nominal income [13].

Since the budget shares sum to unity by definition, the model parameters must satisfy the following adding-up restrictions:

$$\sum_i \alpha_i = 1 \qquad \sum_i \beta_i = 0 \qquad \sum_i \gamma_{ij} = 0 \qquad i = 1, \ldots, n \tag{7}$$

Imposing these restrictions results in one equation of the system to become redundant, implying that one equation may be dropped for estimation. Uncompensated price and income elasticities can be derived from (5) and (6) fairly easily. They are listed below:

$$\varepsilon_{ij} = \frac{1}{w_i}\left[\gamma_{ij} - \beta_i\left(\alpha_j + 0.5\sum_{j=1}^{n}\left(\gamma_{ij} + \gamma_{ji}\right) \ln P_j\right)\right] - \delta_{ij}, \qquad \delta_{ij} = 1 \text{ if } i = j, 0 \text{ otherwise} \tag{8}$$

$$\eta_i = 1 + \frac{\beta_i}{w_i} \tag{9}$$

It should be noted that these elasticities are not constant, since they depend on the evolution of income and prices.

The theory of the rational consumer implies that demand functions are homogeneous of order zero, ensuring that a proportional change in all prices and nominal income does not affect the quantities demanded. Price effects are also expected to be symmetric. The implications of these restrictions on the elasticities are discussed below.

[13] It is customary to refer to X as income, although the variable actually represents total expenditures on all items, i.e. disposable household income minus saving.

Homogeneity restrictions

Imposing homogeneity conditions on model (5) implies the following restrictions on the model parameters:

$$\sum_{j} \gamma_{ij} = 0 \qquad \text{j = 1, ...,n} \tag{10}$$

Equations (5) and (6) change accordingly, and the price elasticities now become:

$$\varepsilon_{ij} = \frac{1}{w_i}\left[\gamma_{ij} - \beta_i\left(\alpha_j + 0.5\sum_{j=1}^{n-1}(\gamma_{ij} + \gamma_{ji}) \ln\left(\frac{P_j}{P_n}\right)\right)\right] - \delta_{ij} \qquad \delta_{ij} = 1 \text{ if } i = j, 0 \text{ otherwise} \tag{11}$$

The income elasticities remain unchanged.

Symmetry restrictions

Symmetry of the price effects can be imposed by setting $\gamma_{ij} = \gamma_{ji}$. The uncompensated price elasticities then simplify to:

$$\varepsilon_{ij} = \frac{1}{w_i}\left[\gamma_{ij} - \beta_i\left(\alpha_j + \sum_{j=1}^{n-1}\gamma_{ij} \ln\left(\frac{P_j}{P_n}\right)\right)\right] - \delta_{ij} \qquad \delta_{ij} = 1 \text{ if } i = j, 0 \text{ otherwise} \tag{12}$$

Note that while the homogeneity restrictions apply to each equation separately, the symmetry restrictions are cross-equation restrictions, which can only be imposed when the share equations (5) are estimated simultaneously.

A linear approximation (LA-AIDS)

A disadvantage of system (5) and (6) in empirical modelling is the fact that it is nonlinear in the parameters. Deaton & Muellbauer suggest using a linear approximation to the price index (6), known as Stone's (1953) price index:

$$\ln P = \sum_{i} w_i \ln P_i \qquad \text{i = 1, ...,n} \tag{13}$$

The approximation appears to work well when the underlying price variables are highly correlated. The obvious advantage of using (13) is that the price index becomes independent of the model parameters, and the model becomes linear. As a result, the expression for the price elasticities simplifies to:

$$\varepsilon_{ij} = \frac{1}{w_i}\left(\gamma_{ij} - \beta_i w_j\right) - \delta_{ij} \qquad \delta_{ij} = 1 \text{ if } i = j, 0 \text{ otherwise} \tag{14}$$

It is a well-known fact that most early empirical applications of the static AIDS model described in this paragraph have generally failed to confirm the homogeneity and symmetry restrictions, apparently refuting the underlying theoretical model of the rational consumer. This was also the case in the original application of Deaton & Muellbauer (1980), who used post-war British data to test their model. The authors offered several possible explanations for this failure, two of which have proved to be of great importance: (i) the static model implies that consumers adjust their budget allocation in the current period, while it is quite likely that expenditure on several items is relatively inflexible in the short run, implying a dynamic adjustment mechanism; (ii) the use of a 'representative household' assumption to ensure exact aggregation over households will be unwarranted when the distribution of household budgets and demographic structure change over time. In subsequent work these complications have led to extensions of the original model, which we discuss in the following paragraphs.

b. The dynamic LA-AIDS model

In their 1980 article, Deaton & Muellbauer suggest to include lagged explanatory variables to capture the sluggishness in the adjustment process to the optimal budget allocation. As a result of subsequent developments in time series methods, however, it has become customary to specify the model in terms of a general error correction model (ECM). This approach allows for short-term dynamic adjustments to suboptimal allocations (which occur because of exogenous shocks to the explanatory variables), while preserving a stable long-run relationship between the structural variables. Assuming that the dynamics of the system can be captured with an autoregressive distributed lag model of order one (ADL(1)), the i-th budget share equation in ECM-form has the following specification:

$$\Delta w_{i,t} = \alpha_i + \sum_i \gamma_{ij}{}^D \Delta \ln P_{j,t} + \beta_i{}^D \Delta \ln\left(\frac{X_t}{P_t}\right) + \lambda_i \left[w_{i,t-1} - \sum_i \gamma_{ij} \ln P_{j,t-1} - \beta_i \ln\left(\frac{X_{t-1}}{P_{t-1}}\right)\right] \tag{15}$$

where λ_i is the usual error correction adjustment parameter, and the long-term equilibrium relationship can be retrieved from the term in square brackets. Homogeneity and symmetry restrictions can be imposed as before, but it should be noted that the dynamic model allows for these restrictions to hold only in equilibrium (in the levels part of the equations). More concretely, the restrictions may be imposed (and tested) on the γ_{ij} but not on the $\gamma_{ij}{}^D$.

c. Heterogeneity in the AIDS model

As pointed out in the previous paragraph, Deaton and Muellbauer (1980) already suggested that the heterogeneity of household characteristics may have been one of the reasons behind the failure of the model to confirm the standard assumptions of microeconomic demand theory empirically. Indeed, the simple AIDS model is only a valid aggregation of underlying household demand functions under rather restrictive conditions about the distribution of household characteristics and their interaction with price and income effects (see Blundell, Pashardes & Weber, 1993). This point can be illustrated easily by considering a simple example: suppose that household spending on, for example, health care increases with the average age of its members, and that this average increases over time due to the ageing of the population. As a result, the aggregate share of health care in the budget will increase over time at given relative prices and income distribution. This uptrend in health spending is likely to be captured by the income variable, whose estimated effect will be biased upward.

Many sources of household heterogeneity are conceivable[14], and it goes without saying that it is quite impossible to incorporate them all in the aggregate consumption model. Past research hints at two possible candidate variables: income inequality and demographic composition (Blundell & Stokes, 2005). The former leads to the inclusion of squared income terms, the latter to the inclusion of variables that measure aspects of household composition such as average size and age. Of these candidates, the age composition of the population appears to be a promising choice: not only is there ample evidence of a strong correlation between age and health care spending (see, e.g., European Economy, 2006), it also seems quite likely that age influences many other consumption decisions [15].

The simplest way to introduce demographic variables in the AIDS model is in log-additive form:

$$w_i = \alpha_i + \sum_i \gamma_{ij} \ln P_j + \beta_i \ln\left(\frac{X}{P}\right) + \delta_i \ln A \tag{16}$$

with the additional restriction that the δ_i parameters sum to zero. The assumption implicit in this specification is that the demographic variable(s) (denoted as A in (16)) do(es) not influence the income effect β_i. While this assumption is somewhat restrictive, it limits the number of additional parameters to be estimated, a considerable advantage when the model is estimated with annual aggregate data.

The empirical model estimated in the next section is the dynamic (ECM) version of equation (16).

2.2.2. Description of the model data

The data used to estimate the demand system described before consist of total consumption expenditures, budget shares, price indices and demographic variables over the period 1980-2010.

The data source for private consumption expenditures is the Classification of Individual Consumption According to Purpose (COICOP) which is used for the national accounts ESA95. Observations cover the period from 1995 to 2010. For the period 1980-1994 retropolations are made based on growth rates of the national accounts ESA79.

Before a model like (16) can be estimated, we must decide which aggregate demographic variable(s) to be used and which aggregates of consumption goods and services to consider. The former decision depends on the way in which demographics are assumed to play a role at the micro-level. In the em-

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14 For instance, households (and their members) differ in terms of income, educational level, and employment status, to mention only the most obvious sources of heterogeneity.

15 Blundell et al. (1993), for example, find significant effects for several age-related variables in their estimated equations for food and alcohol expenditure shares.

pirical application that we describe in this paragraph, we will use the share of people aged between 65 and 74, and of people 75 years and older in the total population. This corresponds with a micro model in which expenditure equations contain two dummy variables corresponding to each age interval, taking the value of one when a household contains at least one member in the interval.

The decision about the consumption categories depends on the level of aggregation of the available data and on assumptions about the separability between groups of these basic commodities. Grouping is unavoidable when more than a few basic consumption categories are available, because the number of parameters grows with the square of the number of categories. It imposes an assumption of separability of the underlying utility functions, implying that the choice between consumption categories of a group depends only on the relative prices of the consumption categories of the group and on total group expenditures, but not on prices of consumption categories of other groups. The separability requirement naturally leads to the use of factor analysis to identify uncorrelated groups of consumption categories. The results suggested the following hierarchical structure: in a first stage the economic agent decides how much to spend on group 1, group 2 and group 3. In a second stage she decides how much of the amount spent on group 1 will be allocated to group 1A, 1B and 1C. These budgets are further allocated over different consumption categories as shown in graph 1.

Graph 1: The Allocation Structure of Total Consumption Expenditures

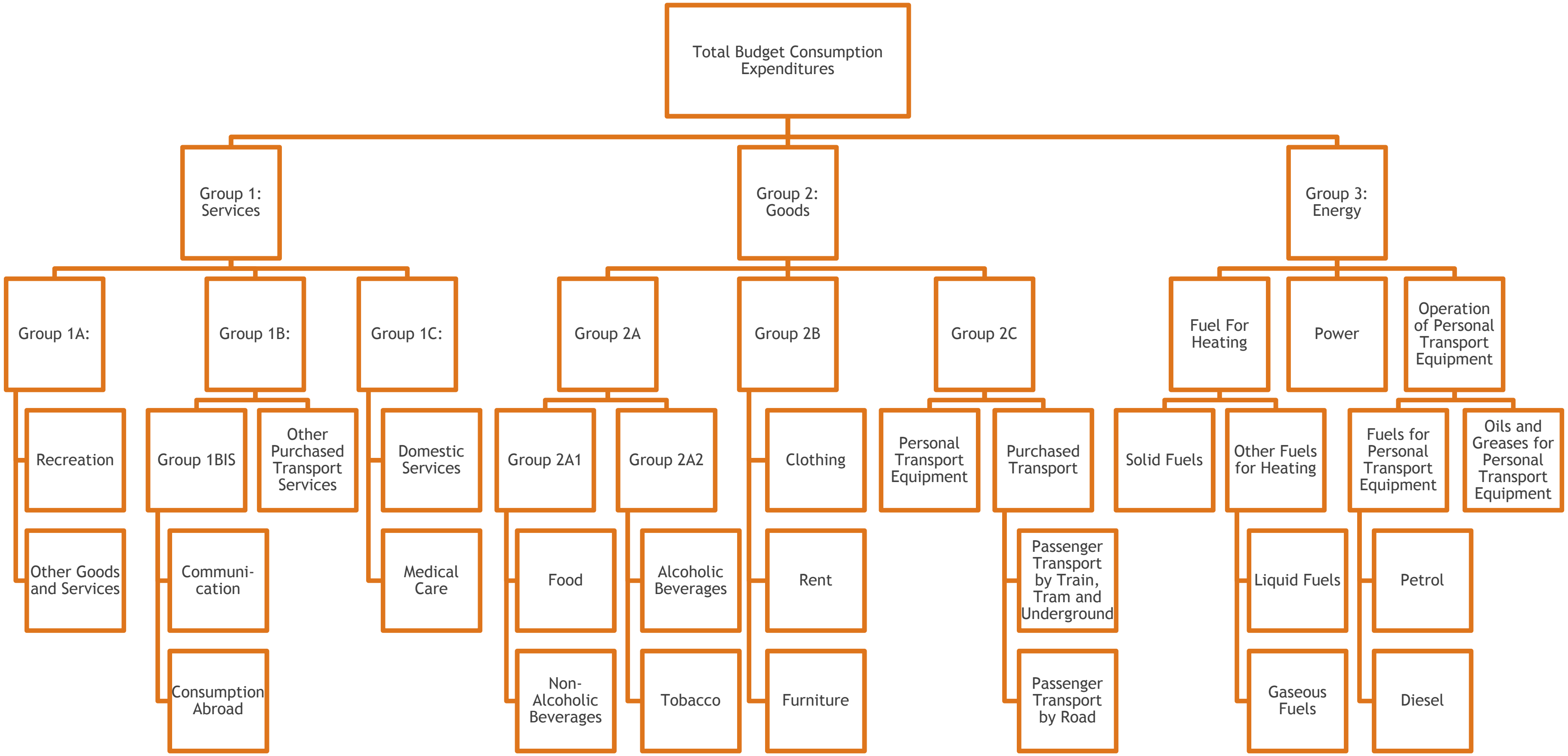


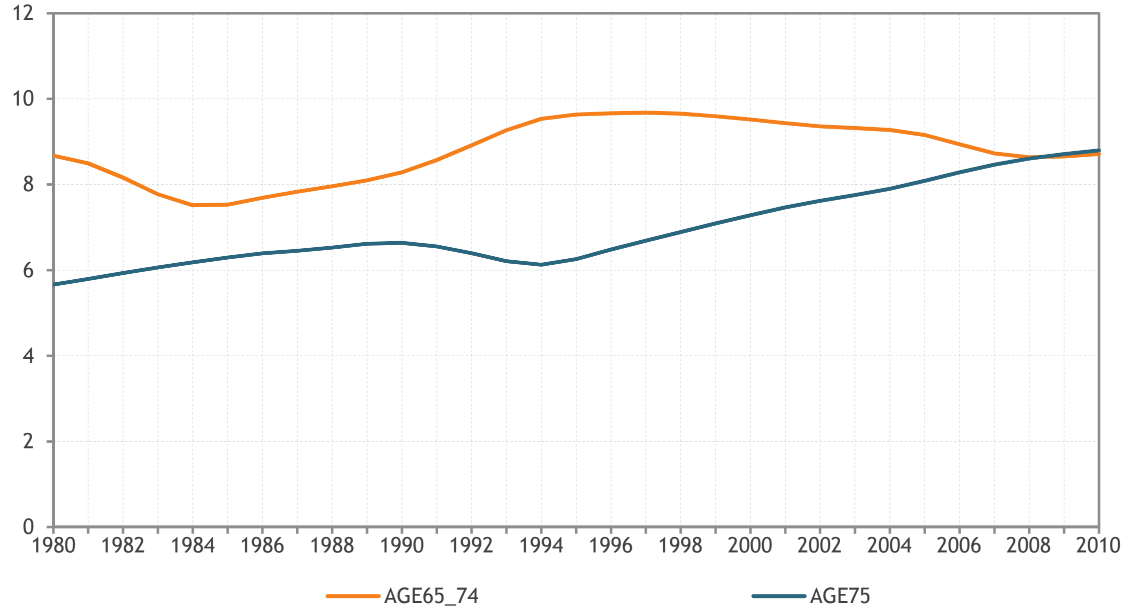
Table 6 presents the "within-group" and the "overall" average budget shares and the standard deviation of the annual growth rates of real consumption expenditures over the period 1980-2010. As we could expect the budget shares of food and rent are highest with an overall budget share for food of 14% on average during the period 1980-2010 and an overall budget share for rent of 15%. The standard deviations indicates that expenditures on energy and particularly on heating grow at a rather volatile rate compared to expenditures on food and rent. Expenditures on personal transport equipment, non-alcoholic beverages, communication, tobacco and medical care grow also at a relatively volatile rate.

Table 6: Within and Overall Average Budget Shares and Standard Deviation of Annual Growth Rates of Real Expenditure

		Within Average Budget Shares 1980-2010	Standard Deviation of Annual Growth Rates of Real Expenditure 1980-2010	Overall Average Budget Shares 1980-2010
1.	**Group 1: Services**	**0.42**	**1.67**	**0.424**
	- ***Group 1A:***	***0.72***	***1.88***	***0.307***
	. Recreation	*0.28*	*3.02*	*0.086*
	. Other Goods and Services	*0.72*	*2.08*	*0.220*
	- ***Group 1B:***	***0.15***	***4.25***	***0.065***
	. Group 1BIS:	*0.96*	*4.37*	*0.062*
	Communication	0.29	5.13	0.018
	Consumption Abroad	0.71	5.49	0.044
	. Other Transport Services	*0.04*	*8.21*	*0.003*
	- ***Group 1C:***	***0.12***	***4.58***	***0.052***
	. Domestic Services	*0.14*	*3.01*	*0.007*
	. Medical Care	*0.86*	*5.39*	*0.044*
2.	**Group 2: Goods**	**0.50**	**1.20**	**0.500**
	- ***Group 2A:***	***0.38***	***1.48***	***0.189***
	. Group 2A1:	*0.81*	*1.75*	*0.153*
	Food	0.93	1.80	0.142
	Non-Alcoholic Beverages	0.07	5.17	0.011
	. Group 2A2:	*0.19*	*2.79*	*0.036*
	Alcoholic Beverages	0.47	2.69	0.017
	Tobacco	0.53	4.10	0.019
	- ***Group 2B:***	***0.54***	***1.27***	***0.270***
	. Clothing	*0.22*	*3.21*	*0.061*
	. Rent	*0.57*	*1.07*	*0.152*
	. Furniture	*0.21*	*2.09*	*0.057*
	- ***Group 2C:***	***0.08***	***5.69***	***0.041***
	. Personal Transport Equipment	*0.80*	*6.82*	*0.033*
	. Purchased Transport	*0.20*	*3.91*	*0.008*
	Transport by Train, Tram, Underground	0.55	5.42	0.005
	Transport by Road	0.45	4.48	0.004
3.	**Group 3: Energy**	**0.08**	**2.73**	**0.077**
	- ***Fuel for Heating***	***0.30***	***6.90***	***0.024***
	. Solid Fuels	*0.06*	*8.80*	*0.001*
	. Other Fuels for Heating	*0.94*	*6.95*	*0.022*
	Liquid Fuels	0.46	7.34	0.011
	Gaseous Fuels	0.54	7.76	0.012
	- ***Power***	***0.30***	***3.24***	***0.022***
	- ***Operation of Personal Transport Equipment***	***0.40***	***2.53***	***0.031***
	. Fuels for Personal Transport Equipment	*0.90*	*2.90*	*0.028*
	Petrol	0.62	3.66	0.017
	Diesel	0.38	3.92	0.010
	. Oils and Greases for Personal Transport Equipment	*0.10*	*15.57*	*0.003*

As already mentioned, two demographic variables, which capture the ageing of the population, are defined as the population aged 65 to 74 and the population aged 75 and over, both expressed as a percentage of the total population. Their evolution over the sample period is shown in graph 2.

Graph 2: Population aged 65-74 and 75+
as a percentage of the total population (1980-2010)



2.2.3. Estimation of the ECM-LA-AIDS model

To estimate the AIDS model described in the previous sections, we have followed a 'general-to-specific' modelling strategy as advocated by David Hendry and others (see for example, Charemza & Deadman, 1992). Taking the general dynamic AIDS model as the starting point (the 'General Unrestricted Model'), we have tested the validity of the following sequence of simplifying restrictions on the model parameters:

- using the linear approximation to the overall price index;
- homogeneity of the demand equations;
- symmetry of the cross-price effects;
- exclusion of non-significant effects.

These restrictions, except the first one, lead to simpler models which are nested in the more general specifications that precede them, so they can be tested with the familiar likelihood ratio (LR) test. The linear price index model, however, cannot be derived from the nonlinear one by imposing restrictions on the latter, nor can both models be conceived as special cases of a common 'parent' model, so neither the LR test nor Hausman's J-test can be used to test the linear approximation. Consequently, either another non-nested formal test should be carried out (such as Vuong's test, 1989; see Greene 2008), or the validity of the approximation should be judged more informally from the estimated log-likelihoods of the alternative models. A final observation is in order: the linear approximation based on the Stone price index, while being the specification commonly used in applied work, is by no means the only possible one. Indeed, recent work (Ogura, 2006) suggests that the Paassche price index may perform just as well, or possibly even better, than the Stone index.

In this section we summarize the major estimation results in the form of the implied long-run overall income and uncompensated price elasticities (see table 7). In order to obtain these elasticities we first check if the allocation model behaves well i.e. we check if it gives stable results when simulating for a long simulation period. For this purpose we simulate the allocation module alone - i.e. not with the complete HERMES model - up to horizon 2100 keeping prices, total consumption expenditure and demographic variables constant. The adjustment process brings the system to a steady state. Next we introduce a 1 percent increase in total real consumption expenditure or in individual prices of consumption categories (period 2050). In the absence of further shocks the error correction mechanism will pull the system to a new steady state. In period 2050 the deviations from the baseline can be interpreted as short-run income and uncompensated price elasticities, while the responses in period 2059 approximate the long-run elasticities. These 'equilibrium' elasticities obtained by simulating the allocation module alone differ from 'real world 'elasticities because of the absence of feedback mechanisms.

Table 7: Long-run Overall Own Uncompensated Price and Income Elasticities

	Long-run Overall Own Uncompensated Price Elasticity	Long-run Overall Income Elasticity
Group 1: Services	**-0.36**	**1.45**
- Group 1A:	***-0.34***	***1.99***
. Recreation	*-0.71*	*1.99*
. Other Goods and Services	*-0.48*	*1.99*
- Group 1B:	***-0.45***	***0.19***
. Group 1BIS:	*-0.47*	*0.19*
Communication	-0.03	0.30
Consumption Abroad	-0.10	0.14
. Other Transport Services	*-0.97*	*0.19*
- Group 1C:	***0.08***	***0.27***
. Domestic Services	*-0.36*	*0.39*
. Medical Care	*-0.01*	*0.24*
Group 2: Goods	**0.00**	**0.49**
- Group 2A:	***-0.13***	***0.36***
. Group 2A1:	*-0.40*	*0.47*
Food	-0.36	0.45
Non-Alcoholic Beverages	-0.15	0.72
. Group 2A2:	*-0.91*	*0.00*
Alcoholic Beverage	-0.73	0.00
Tobacco	-1.01	0.00
- Group 2B:	***-0.20***	***0.48***
. Clothing	*-0.91*	*1.38*
. Rent	*0.01*	*0.08*
. Furniture	*-0.22*	*0.87*
- Group 2C:	***-1.23***	***0.90***
. Personal Transport Equipment	*-1.37*	*1.05*
. Purchased Transport	*-0.85*	*0.07*
Transport by Train, Tram, Underground	-0.40	0.09
Transport by Road	-0.54	0.05

	Long-run Overall Own Uncompensated Price Elasticity	Long-run Overall Income Elasticity
Group 3: Energy	**-0.44**	**1.36**
- Fuel for Heating	***-0.15***	***1.78***
. Solid Fuels	*-0.97*	*1.78*
. Other Fuels for Heating	*-0.17*	*1.78*
Liquid Fuels	-0.17	1.42
Gaseous Fuels	-0.11	1.99
- Power	***-0.31***	***1.27***
- Operation of Personal Transport Equipment	***-0.34***	***0.98***
. Fuels for Personal Transport Equipment	*-0.41*	*0.98*
Petrol	-0.32	0.98
Diesel	-0.41	0.98
. Oils and Greases for Personal Transport Equipment	*-0.92*	*0.98*

In former estimations (see Bracke and Willemé (2012)), we obtained rather high long-run overall price elasticities for tobacco, while in the literature (Chaloupka, 2002) reported long-run price elasticities do not exceed -1. Consequently, we have imposed (and tested) restrictions on some of the coefficients of the consumption categories in group 2A in order to reduce the implied own price elasticity of tobacco. The new long-run elasticity is substantially less elastic and in line with the literature.

The implied long-run income elasticities are relatively high for recreation, other goods and services, clothing, personal transport equipment and energy consumption categories. Communication, consumption abroad, domestic services, medical care, food, alcoholic beverages, tobacco, rent and purchased transport have rather low long-run income elasticities.

The long-run uncompensated own price elasticities, who take into account the income effect of a price change, are relatively low for communication, consumption abroad, domestic services, medical care, food, non-alcoholic beverages, rent (positive sign for but non-significantly different from zero), furniture, purchased transport and energy consumption categories. Alcoholic beverages, tobacco, clothing and personal transport equipment show rather high price elasticities.

2.3. Housing investment

Residential investment at constant prices (IRO) is also modelled by means of an ECM, assuming a long term relationship with real disposable income (YDH/PCH) and a mortgage rate indicator (RMR). It is assumed that residential investment has a lagged response to the evolution of its explanatory variables. The long run equation for housing investment can be written as:

$$\ln IRO_t = \alpha \ln \left(\frac{YDH}{PCH}\right)_{t-i} + \beta \ln RMR_{t-i} \qquad i = 0, \ldots 2 \tag{17}$$

The long-run elasticity of housing investment to real income is not imposed and is estimated econometrically. Intuitively, we can expect a largely positive elasticity (between 0.5 and 1.0). The long-run effect of interest rate should be largely negative; this parameter is also econometrically estimated.

Both determinants also appear in the short-term relationship, together with the evolution of the unemployment rate UR (a confidence indicator) and the price level of housing investment PIR with respect to the consumption price level PCH (a decreasing relative price makes housing investment more attractive). Short term determinants can be also lagged. This gives the following relationship:

$$\begin{aligned}\Delta \ln IRO_t = {} & [\alpha + \beta\, \Delta \ln \left(\frac{YDH}{PCH}\right)_t + \gamma\, \Delta \ln RMR_{t-1} + \delta\, \Delta \ln IRO_{t-1} + \varphi\, \Delta \ln \left(\frac{PIR}{PCH}\right)_t \\ & + \eta\, \Delta \ln UR_t\,] + \lambda\, [\ln IRO_{t-1} - \theta \ln \left(\frac{YDH}{PCH}\right)_{t-1} - \mu \ln RMR_{t-2}\,]\end{aligned} \tag{18}$$

In equation (18), β (the short-term coefficient of real disposable income) should be positive (β≤1), γ (the short term coefficient of the interest rate) should be negative, as well as φ and η (respectively the short-term parameters of relative prices and unemployment).

In the long-term relationship, λ, the error correction adjustment parameter (ECM), must be negative, θ (the long term parameter of the real disposable income) must be positive and close to 1.0, and μ (the parameter of the interest rate) is expected to be largely negative.

The estimation results (on the period 1987-2011) for equation (18) are the following:

Table 8: Coefficients of the housing investment function

Coefficient	α	β	γ	δ	φ	η	λ	θ	μ
Estimation value	0.0	0.37	-0.45	0.32	-0.40	0	-0.30	0.81	-1.68
t-test	(-)	0.37	-0.20	1.48	-0.37	(-)	-3.26		
Other tests	Adjusted R^2	F-Stat	Durbin-Watson	Log-likelihood					
	0.42	3.83	2.34	40.66					

Note that all coefficients have the correct expected sign but are, sometimes, not significantly different from 0. The introduction of unemployment together with the other explanatory variables didn't give significant results.

Elasticities of housing investment

As can be seen from table 9, an increase of 1% of the real disposable income increases the housing investment with 0.82% after 4 years and 0.87% in the long run. Moreover, the econometric estimation confirms that the interest rate has a large negative effect on housing investment: if the effect of an interest rate increase equal to 100 basis points does not affect housing investment the first year, we observe a negative effect close to 1.3 after 4 years and nearly equal to 2 in the long term.

Table 9: Housing investment elasticities

	t	t+3	t+5	Long term
Real disposable income	0.37	0.82	0.87	0.87
Interest rate	0.00	-1.34	-1.66	-1.69

2.4. Computation of the disposable income of households

We saw that disposable income of households (YDH) is the main determinant of private consumption and housing investment. This point briefly describes how YDH is computed in HERMES.

2.4.1. Total disposable income of households

Aggregated disposable income of households is computed using the following identity:

$$YDH = REMI + WBU + YN + GOSH + IDH + SBH + OCUH - (DTOTH + DTHUE + YSSH) \tag{19}$$

where:

$REMI$ is mixed income (income of non wage workers);

WBU is total compensation of employees;

YN is compensation of cross-border workers;

$GOSH$ is gross operating surplus of households;

IDH is net property income of households;

SBH are social benefit receipts of households;

$OCUH$ are net other current transfers of households;

$DTOTH$ is income tax paid by households to the Belgian general government;

$DTHUE$ is income tax paid by households to the European Union;

$YSSH$ are social security contributions paid by households.

2.4.2. Components of the disposable income of households

a. Mixed income

Mixed income $REMI$ depends on the evolution of the number of self-employed and of the average gross wage per capita.

$$\begin{aligned} d \ln REMI_t = {} & \alpha\, d \left(\beta + \gamma \ln\left(NI * \left(\frac{WBFU}{NF}\right)\right)\right) + \epsilon \left(\beta + \gamma \ln\left(NI * \left(\frac{WBFU}{NF}\right)\right)_{t-1}\right) \\ & - \epsilon \ln REMI_{t-1} \end{aligned} \tag{20}$$

where:

NI is the number of self-employed;

$WBFU$ is the gross payroll (market industries);

NF is the number of employees (market industries).

b. Compensation of employees

WBU, which represents the total wage bill (including wage subsidies, cf. point 4.3.4), is the sum of wages paid by every market and non market branch s:

$$WBU = \sum_{s} WB_s + LSUB_s \tag{21}$$

where WB_s is the compensation of employees paid by industry s and computed by age group (workers younger or older than 50 years old) and by wage level (workers with a high or a low level of wage).

c. Compensation of cross-border workers

YN, which is the compensation of cross-border workers, is computed as the difference between the compensation of Belgian residents working abroad (YNX) and the compensation of not Belgian residents working in Belgium (YNM). YNX and YNM evolve with, respectively, the number of cross-border workers going out of Belgium (EFX) and the number of cross-border workers entering Belgium (EFM). It is assumed that both those types of cross-border workers earn the average wage paid by Belgian firms.

$$dln\,(YNM) = dln\left(EFM * \left(\frac{WBF}{(NF + NDOM)}\right)\right) \tag{22}$$

$$dln\,(YNX) = dln\left(EFX * \left(\frac{WBF}{(NF + NDOM)}\right)\right) \tag{23}$$

d. Gross operating surplus

$GOSH$, gross operating surplus of households, is computed as

$$GOSH = QVUFH - (REMI + WBFH + ITPDH - SUBDGH) \tag{24}$$

where:

$QVUFH$ is the value added of households (a fraction of total firms value added);

$WBFH$ is the compensation of employees paid by the households (including wage subsidies $LSUB$):

$$WBFH = \sum_{s} \alpha_s\,(WB_s + LSUB_s) \tag{25}$$

where s = industries;

$ITPDH$ are taxes on production paid by households;

$SUBDGH$ are subsidies on production paid by public administrations to households.

e. Property income

IDH, net property income of household, is the sum of three components:

$$IDH = IDHI + IDHD + IDHR \tag{26}$$

where:

$IDHI$ represents the net interest payments;

$IDHD$ represents the dividends distributed by enterprises;

$IDHR$ represents the net other property income (notably insurance income).

f. Social benefit receipts of households

The sum of all social transfers paid by the different institutional sectors to the households (SBH) is computed as follows:

$$SBH = SBG + SBF + SBH\_B + SBI + SBB\_B - SBB\_M \tag{27}$$

where:

SBG represents the total allowances paid to the households by the public administrations (see also Chapter 6 devoted to public finances):

$$SBG = \sum_i SBG_i \tag{28}$$

where i= the different levels of general government;

SBF represents the allowances paid to the households by the enterprises;

SBH_B represents imputed social allowances;

SBI represents the social allowances paid to the households by the non profit institutions;

SBB_B represents social allowances paid by other countries;

SBB_M represents social allowances paid to other countries.

g. Other current transfers of households

$OCUH$ represents the net (received – paid) other current transfers of households. This aggregate has been negative since 1995. It includes insurance premiums, fines, ...

h. Income tax paid to the Belgian general government

$DTOTH$, the income tax, is computed in the public finances block (see also chapter 6). $DTOTH$ is equal to:

$$DTOTH = DTH + TCP + DTO4 + DTPAT2 + DTDIV2 + TITARX\_H \tag{29}$$

where:

DTH is the personal income tax paid by households;

TCP are the taxes on traffic paid by households;

$DTO4$ is the sum of the Walloon Region's domestic waste tax and of the Brussels Capital Region's flat-rate tax;

$DTPAT2$ are taxes on patrimony (grounds and buildings) paid to the local government;

$DTDIV2$ represents other taxes on income;

$TITARX\_H$ are securitisations (physical persons, 2005 and 2006).

i. Income tax paid to the European Union

$DTHUE$ is the income tax paid by EU officials who live in Belgium to the European Union.

j. Social security contributions

The total social security contributions paid by households, $YSSH$, is the sum of the following variables:

$$YSSH = SSF + SSH + SSB\_M - SSB\_B \tag{30}$$

where:

SSF represents the employers' social security contributions;

SSH represents the employees' social security contributions;

SSB_M represents the social security contributions paid by Belgian workers working for European institutions or abroad;

SSB_B represents the social security contributions paid by foreigners.

3. External trade

The external trade block is a crucial part of the HERMES model, given the high degree of openness of the Belgian economy. It contains equations of import volumes, import prices, export volumes and export prices by industry.

For both exports (section 3.1) and imports (section 3.2), the specifications of the equations of volumes and prices are described in a first sub-section (3.1.1 and 3.2.1 respectively). Then the results of the estimation of those equations are presented in a second sub-section (3.1.2 and 3.2.2 respectively).

3.1. Exports

3.1.1. Specification

The theoretical point of departure consists of a system of simultaneously-estimated equations explaining the evolution of export volumes and prices by distinguishing the long-term and short-term behaviour within the framework of an error correction model.

a. Volumes

The modelling integrates two traditional export determinants: the world demand of goods and services and the price competitiveness of national producers. Supply effects are also tested by introducing production costs in the equation (see below).

The tested base specification is as follows:

$$\begin{aligned}\Delta \ln QXO_i = {} & \alpha + \beta\, \Delta \ln QWX + \gamma\, \Delta \ln \left(\frac{PQX_i}{\frac{PWX}{EXRAT}} \right) + \delta\, \Delta \ln \left(\frac{PQX_i}{PB_i} \right) \\ & + \lambda \left(\ln\left(QXO_{i,t-1}\right) - \theta \ln \left(QWX_{t-1} - \mu \ln \left(\frac{PQX_i}{\frac{PWX}{EXRAT}} \right)_{t-1} - \tau \ln \left(\frac{PQX_i}{PB_i} \right)_{t-1} \right) \right)\end{aligned} \tag{31}$$

The underlying long-term relation is as follows:

$$QXO_i^* = f\left[QWX, \frac{PQX_i}{\frac{PWX}{EXRAT}}, \left(\frac{PQX_i}{PB_i}\right) \right] \tag{32}$$

Exports by industry i (QXO_i) depend on a world demand indicator (QWX), a competitiveness indicator which is defined as the ratio between export prices (PQX_i) and world prices (PWX expressed in euro using the exchange rate $EXRAT$) and a margin indicator defined as the ratio of export prices to domestic production costs (PB_i).

Generally speaking, the long-term specification is double logarithmic: all variables, dependent and explanatory, are expressed in logarithms and the coefficients can be read directly as elasticities. For some industries, however, the long-term adjustment is markedly better, from the perspective of the sign (negative or positive) and significance of the coefficients of the explanatory variables – and in particular price elasticity – when one adopts a semi-logarithmic relation (especially whenever QWX is expressed in level, not in logarithm).

The world demand indicator is defined as an index of potential markets for Belgium, calculated as the weighted average of the expected import growth of Belgium's trading partners. This indicator is not available by industry, but only at the aggregate level.

World prices (PWX) are calculated as a weighted average of the import prices of Belgium's main trading country-partners. This indicator, too, is not available per branch of activity.

For the coefficients, the following signs are expected:

– β, $\theta > 0$ (world demand effect): an increase of world demand (QWX) should boost national exports;
– γ, $\mu < 0$ (price competitiveness effect): if world prices ($PWX/EXRAT$) grow stronger than export prices PQX_i, a larger share of world demand should move to national products other things being equal;
– δ, $\tau > 0$ (margin effect PQX_i/PB_i): if domestic production costs (PB_i) decrease, all other things remaining equal, profits on external markets should increase, resulting in rising exports (supply effect);
– $\lambda < 0$: a negative coefficient of the lagged error term expresses a catching-up force of export volumes towards the long-term relation.

We have also tested the introduction of a trend (linear or quadratic) in the export volume equations in order to capture possible structural losses of market shares (cf. Verschueren, 2009). However, this trend only proved statistically significant for "equipment goods" (K).

b. Prices

The tested base specification is as follows:

$$\Delta \ln PQX_i = \alpha + \beta\, \Delta \ln\left(\frac{PWX}{EXRAT}\right) + \gamma\, \Delta \ln PB_i + \lambda \left[\ln PQX_{i,t-1} - \epsilon \ln \left(\frac{PWX}{EXRAT}\right)_{t-1} - (1-\epsilon) \ln PB_{i,t-1}\right] \tag{33}$$

The underlying long-term relation is as follows:

$$PQX_i^* = f\left[\frac{PWX}{EXRAT}, PB_i\right] \tag{34}$$

In the HERMES model, export prices by industry i are determined endogenously by domestic production costs (PB_i) and world prices expressed in euro ($PWX/EXRAT$).

The two short-term coefficients (β and γ) are expected to be positive: Belgian export prices should follow domestic costs (even more so if exporters are "price setters", i.e. they have a certain degree of market power) as well as world prices (even more so if exporters are "price takers" on foreign markets).

Moreover, as it is common in literature[16], we only constrain the sum of the two price coefficients to 1 for the long term: in the long run, growth of Belgian export prices is a weighted average of the respective growth rates of these two price factors.

Coefficient ε is expected to be positive since, in general, Belgian enterprises do not have a monopoly power that would allow them to fix their export prices merely on the basis of their own production costs and the demand for their products (without considering the prices applied by foreign producers).

Coefficient λ is expected to be negative since any deviation of export prices from the long-run relationship is corrected, more or less quickly (in the subsequent years).

3.1.2. Estimation results

a. Preliminary remarks

The econometric assessments are carried out at the level of the HERMES industries. In practice, they include seven industries: "agriculture" (A), "construction" (B), "consumer goods" (C), "equipment goods" (K), "other market services (than transport and communication)" (L), "intermediate goods" (Q) and "transport and communication" (Z).

As for the latter branch, estimations are achieved at this aggregated sectoral level and not by sub-branch ("rail and road transportation" (Z_12), "air, maritime and fluvial transportation" (Z_34), "other transport services and communication" (Z_56)).

[16] See, for instance, Allard-Prigent et al (2002).

That also applies to the other market services branch: the branch L corresponds to the sum of the branches “trade, lodging and catering services” (HA), “services of credit and insurance institutions” (CR), “health care” (SA) and “other market services to households and firms” (OS).

The estimation period is either 1981-2011 or 1986-2011.

Finally, export volumes and prices of the branch “energy” (E) are determined in a specific manner within the energy block of the model. They are subject to a detailed assessment by energy product (see point 5.4.2).

We immediately note that the estimation of those equations is far from simple for several reasons:

– export and import series (prices and volumes) by NACE industry are not published by the National Accounts Institute in the context of the National Accounts; more specifically, they only distinguish between goods, services and "tourism"; hence, we built our own data for the purpose of econometric estimation;
– we only dispose of indicators of world demand (QWX) and world prices (PWX) at an aggregate level, not by industry;
– export statistics include re-exports of goods not of Belgian origin (imported goods which leave the country again practically unprocessed or in the same state as previously imported), a phenomenon that has spread over the past years, Belgium being a country in which large quantities of goods transit (in particular through the port of Antwerp). Yet, those re-exports are clearly not affected by the same factors as “traditional” Belgian exports; they are markedly less influenced by price or domestic cost type variables than by other factors such as the existence of major ports, developed infrastructure, the proximity of large countries, ...

Two estimation methods were used: (1) a joint assessment of both the short- and the long-term relationships, and (2) a two-step ECM procedure in which we test, in a first-step, different functional forms for the long-run relation, as already mentioned earlier.

Note that some variables for which the estimated coefficient showed the expected sign but was not significant (at 15%) were maintained in the final specification.

b. Volumes

Table 10 presents the results of the assessment of export volume equations. It shows the estimated parameters, and for each of them, the value of the Student test statistic (t-test) and in the three last lines, the value of the adjusted R^2, of the Durbin-Watson test statistic (test of serial autocorrelation of rank 1 errors) and of the Fisher test statistic (F-test).

For the sake of comprehensiveness, we note that a binary variable for the year 1995 was added in some equations (branches Q, B and L) and a quadratic trend was included in the equation of exports of "equipment goods" (K).

Table 10: Estimation results of export equations - Volumes

EXPORT VOLUMES	A	B	C	K	L	Q	Z
Constant	1.38	1.24	1.89	7.98	2.46	4.36	3.23
	(1.50)	(1.26)	(1.94)	(3.63)	(1.84)	(2.59)	(2.17)
Short-term elasticities							
QWX	0.49	0.66	0.58	1.10	0.29	1.22	0.49
	(1.83)	(2.12)	(4.62)	(6.08)	(1.18)	(9.28)	(1.81)
$PQX_i/(PWX/EXRAT)$	0**	-0.06	-0.34	-0.52	-0.15**	-0.40	-0.83
		(-0.16)	(-1.84)	(-1.97)		(-1.56)	(-1.99)
PQX_i/PB_i	-	0.20	0.09	-	-	-	0.41
		(0.31)	(0.28)				(1.15)
ECM							
$QXO_i[-1]$	-0.17	-0.18	-0.17	-0.88	-0.23	-0.39	-0.34
	(-1.50)	(-1.25)	(-1.93)	(-3.80)	(-1.81)	(-2.61)	(-2.16)
Long-term elasticities							
QWX[-1]	0.50**	0.85	0.70**	1.14	0.91	0.89	0.85
$(PQX_i/(PWX/EXRAT))[-1]$	-0.11*	-0.39*	-0.38*	-0.78	-0.63	-0.72*	-1.42
$(PQX_i/PB_i)[-1]$	-	1.57	0.30**	-	0.60	-	0.45
Adjusted R^2	0.14	0.47	0.53	0.80	0.64	0.82	0.38
Durbin Watson	2.32	1.59	1.58	1.79	2.23	1.58	1.56
F-test	2.98	4.11	8.08	15.26	8.34	24.14	3.15

* Parameter from a separately-estimated long-term relation.

** The value of the coefficient was fixed during the estimation.

A = agriculture, B = construction, C = consumer goods, K = equipment goods, L = other market services (than transport and communication), Q = intermediate goods, Z = transport and communication.

Both the short-term and the long-term elasticities of export volumes with respect to world demand (QWX) are lower than 1 for all service branches ("transport and communication" (Z), and "other market services" (L)), "construction" (B), "agriculture" (A) and the branch "consumer goods" (C).

For the two industries most exposed to international competition – i.e. "intermediate goods" (Q) and "equipment goods" (K), they are higher than 1 in the short term and reach 0.89 and 1.14 respectively in the long term.

The elasticities of export volumes to the price competitiveness indicator ($PQX_i/(PWX\ /EXRAT)$) are relatively low, except for "transport and communication" (Z) both in the short and the long run.

The majority of long-term price elasticities were assessed separately and not together with the short-term relation.

Specifically, nearly no margin effect (PQX_i/PB_i) on export quantities was found in both the short and the long term (even when the specification did not include the variable $PQX_i/(PWX/EXRAT)$). The coefficient of that indicator was set to 0 for nearly all industries; it is 0.4 for Z (with a t-stat of 1.2) and it is weak (not statistically significant), though being maintained, for B and C.

The error correction coefficients in the volume equations in table 10, with the exception of that of "equipment goods" (K), show values between -0.17 and -0.39, depending on the branch of activity. The adjustment towards the long-term relationship would be made after three to five years.

c. Prices

Table 11 presents the estimation results for the export prices equations. It shows the estimated parameters, the value of the Student test statistic (*t*-test) for each one of them, the adjusted R^2, the Durbin-Watson test statistic and the Fisher test statistic (F-test).

Table 11: Estimation results of export equations - Prices

EXPORT PRICES		A	B	C	K	L	Q	Z
Constant		0.01	-0.00	-0.00	0.00	0.00	-0.02	0.01
		(1.31)	(-0.60)	(0.62)	(0.47)	(0.31)	(-2.91)	(1.91)
Short-term elasticities								
	PB_i	0.06	0.59	0.40	0.20	0.33	0.56	0.06
		(0.65)	(2.21)	(2.34)	(1.27)	(1.08)	(4.32)	(0.55)
	PWX/EXRAT	0.46	0.30	0.27	0.17	0.45	0.40	0.24
		(4.05)	(3.10)	(2.44)	(1.84)	(3.97)	(3.06)	(2.10)
ECM								
	PQX_i[-1]	-0.34	-0.22	-0.21	-0.26	-0.35	-0.34	-0.14
		(-2.60)	(-1.81)	(-2.20)	(-3.86)	(-1.98)	(-3.19)	(-1.42)
Long-term elasticities								
	PB_i[-1]*	0.43	0.38	0.00	0.00	0.68	0.00	0.49
	(PWX/EXRAT)[-1]	0.57	0.62	1.00**	1.00**	0.32	1.00**	0.51
Adjusted R^2		0.44	0.36	0.43	0.33	0.42	0.80	0.11
Durbin-Watson		2.16	1.73	1.33	1.61	1.72	1.80	1.34
F-Test		5.82	4.56	10.32	5.14	5.61	34.43	1.74

* For any industry, the value of the parameter is equal to 1-coeff PWX.
** The value of the coefficient was fixed during the estimation.
A = agriculture, B = construction, C = consumer goods, K = equipment goods, L = other market services (than transport and communication), Q = intermediate goods, Z = transport and communication.

In the short run, export price setting is fairly balanced between margin and competitiveness behaviour, although more driven by international prices in the branches "agriculture" (A) and "transport and communication" (Z) and by domestic costs in "construction" (B).

In the long run, in the three manufacturing industries (Q, K and C), Belgian export prices are entirely determined by world prices, suggesting a behaviour exclusively guided by competitiveness imperatives in those industries. Indeed, the long-term world price coefficient was significantly higher than 1 in initial econometric estimates. In this case, we chose to set the world price coefficient to 1 (and the domestic cost coefficient to 0). Belgian exporters of manufacturing goods would behave as "price takers" in the long run – which appears consistent with the size and openness of the Belgian economy.

In the “other market services” (L), on the contrary, exporters would rather act as “price setters” in the long run whereas a behaviour shared between world prices and domestic costs considerations would prevail in the branches “transport and communication” (Z) and “agriculture” (A).

The error correction mechanism seems to be fairly similar in export price and export volume equations (any deviation from the long-term relationship would be corrected in three to five years according to the branch), except for the “equipment goods” industry (K).

d. Elasticities

Table 12 shows the aggregated (all industries except “energy”) reaction of exports following a 1% increase in the potential foreign demand addressed to Belgium. The sensitiveness of exports to a 1% depreciation of the euro (increased export price competitiveness of Belgium) is also displayed.

Table 12: Export elasticities

	t	t+3	t+5	Long term
Potential world demand	0.86	0.86	0.88	0.90
Competitiveness	0.40	0.61	0.65	0.68

The elasticity of exports to the potential world demand addressed to Belgium appears to be constant over time: a 1% increase of potential foreign demand would increase Belgian exports by 0.86% in the short term and by 0.90% in the long term. Below-unity elasticities of exports to potential world demand imply market share losses for Belgium.

The increase of exports following improved price competitiveness would be more progressive: coming from a rise of 0.40% in year t it would progressively increase to 0.68% in the long term.

3.2. Imports

3.2.1. Specification

Again, the theoretical point of departure consists of a system of simultaneously-estimated equations explaining the evolution of import volumes and prices by distinguishing the long term and short term behaviour within the framework of an error correction model.

As for exports, the variables are modelled in logarithms in most cases.

a. Volumes

The tested base specification is as follows:

$$\begin{aligned}\Delta \ln QMO_i = {} & \alpha + \beta\, \Delta \ln D_i + \gamma\, \Delta \ln\left(\frac{PQM_i}{PQ_i}\right) \\ & + \lambda \left[\ln QMO_{i,t-1} - \epsilon \ln D_{i,t-1} - \theta \ln\left(\frac{PQM_i}{PQ_i}\right)_{t-1}\right]\end{aligned} \qquad (35)$$

Where:

$$D_i = QQO_i + QHO_i + QXO_i + SO_i + QIO_i + QGO_i \tag{36}$$

The underlying long-term relationship is as follows:

$$QMO_i^* = f\ [D_i\ , PQM_i\ /\ PQ_i] \tag{37}$$

Generally speaking, the long-term specification is double-logarithmic: all variables, dependent and explanatory, are expressed in logarithms and the coefficients can be read directly as elasticities.

The import volume QMO_i of products similar to those produced by industry i depends on total demand (both domestic and foreign) for products of industry i (D_i) and a index of price competitiveness defined as the ratio between import prices (PQM_i) and domestic production prices (PQ_i). Note that we initially wanted to consider supply effects by taking the capacity utilization rate (QR_i) into account. However, this variable was finally omitted because its coefficient did not have the expected sign or was not significantly different from zero.

The total demand addressed to any industry i is defined as the sum of the deliveries of this industry to intermediate consumption (QQO_i), to private consumption (QHO_i), to exports (QXO_i), to changes in inventories (SO_i), to gross fixed capital formation (QIO_i) and to public consumption (QGO_i).

Note that we initially intended to distinguish between a short-term elasticity of imports to intermediate demand (QQO_i), on the one hand, and to the rest of final demand ($QHO_i + QXO_i + SO_i + QIO_i + QGO_i$), on the other hand. However, this proved practically possible for only two industries: "consumer goods" (C) and "intermediate goods" (Q) (see table 13 below). For the other industries, the coefficient of intermediate demand was not significantly different from zero. That is why we decided to add intermediate demand to the other categories of demand so that only one coefficient for total demand was estimated. The latter always turned out to be significant. Note that for "transport and communication" (Z) we chose to distinguish, in the short term, the internal demand ($QQOZ + QHOZ + SOZ + QIOZ + QGOZ$) addressed to this industry from the external demand ($QXOZ$). The effect of a 1% change in each of both components on imports is indeed differentiated.

The signs of the coefficients are expected as follows:

– β, ε > 0 (demand effect): an increase in the demand (D_i) addressed to the industry should result in an increase in the volume of imported goods required to meet this additional demand;

– γ, θ < 0 (price competitiveness): the higher the domestic production price (PQ_i) relative to the import price (PQM_i), the higher the imports of products similar to those produced by Belgian producers (industry i);

– λ < 0: the catching-up force of import volumes toward the long-term relationship must be revealed by a negative estimated coefficient of the error correction term.

b. Prices

The tested base specification is as follows:

$$\Delta \ln PQM_i = \alpha + \beta \, \Delta \ln \left(\frac{PWM}{EXRAT}\right) + \gamma \, \Delta \ln PQ_i + \lambda \left[\ln PQM_{i,t-1} - \epsilon \ln \left(\frac{PWM}{EXRAT}\right)_{t-1} - (1-\epsilon) \ln PQ_{i,t-1}\right] \quad (38)$$

The underlying long-term relationship is as follows:

$$PQM_i^* = \mathrm{f}\left[\frac{PWM}{EXRAT}, PQ_i\right] \quad (39)$$

In the HERMES model, import prices are determined endogenously by domestic production prices (PQ_i) and world prices expressed in euro ($PWM/EXRAT$). PWM is computed as the weighted sum of export prices of Belgium's main trading partners. This indicator is only available at the aggregate level, not by industry.

The parameters in the base specification above have the following expected signs:

- β, ε > 0: Belgian import prices should, at least partially, follow the evolution of world prices expressed in euro ($PWM/EXRAT$); the value of ε allows us to measure the respective influences of internal and external prices on import prices in the long run. Whenever ε equals zero, the import price only depends on internal prices in the long term; whenever ε equals one, the imported price is fully-indexed to world prices in the long term;
- $\gamma > 0$: Belgian import prices should also partially reflect the evolution of Belgian domestic production prices (PQ_i);
- $\lambda < 0$: it is expected that any deviation of import prices from the long-term relationship is corrected, more or less quickly (in the subsequent years).

We constrain the sum of the coefficients of both world prices and domestic production prices to unity in the long run (this is common in literature[17]): the log of Belgian import price (PQM_i) is then a weighted average of those two price factors (in logarithms).

3.2.2. Estimation results

a. Preliminary remarks

The preliminary remarks made for exports (see sub-section 3.1.2.a) also apply here.

[17] See, for instance, Allard-Prigent et al (2002).

b. Volumes

Table 13 presents the results of the estimation of import volume equations. It shows the estimated parameters, the respective associated Student test statistics (*t*-test), the value of the adjusted R^2, of the Durbin-Watson test statistic (test of serial autocorrelation of rank 1 errors) and of the Fisher test statistic (F-test).

For the sake of comprehensiveness, note that time dummy variables were added in some equations (branches A, B, K, L and Z).

Table 13: Estimation results of import equations - Volumes

IMPORT VOLUMES	A	B	C	K	L	Q	Z
Constant	-0.71	1.89	-1.96	-0.55	2.44	-1.14	0.78
	(-2.14)	(3.64)	(-1.88)	(-1.52)	(2.11)	(-4.04)	(57.56)
Short-term elasticities							
QQO_i	-	-	0.42	-	-	0.65	-
			(2.17)			(3.44)	
$(QHO_i+QXO_i+SO_i+QIO_i+QGO_i)$	-	-	0.87	-	-	0.55	-
			(5.38)			(4.99)	
QXO_i	-	-	-	-	-	-	0.99
							(6.55)
$(QQO_i+QHO_i+SO_i+QIO_i+QGO_i)$	-	-	-	-	-	-	0.62
							(1.65)
$(QQO_i+QHO_i+QXO_i+SO_i+QIO_i+QGO_i)$	1.33	0.72	-	1.01	0.92	-	-
	(4.51)	(3.44)		(14.89)	(2.09)		
(PQM_i/PQ_j)	0**	0**	-0.66	-0.29	-0.46	-0.74	0**
			(-3.13)	(-1.95)	(-1.19)	(-2.78)	
ECM							
QMO_i [-1]	-0.26	-0.30	-0.31	-0.10	-0.29	-0.46	-0.10**
	(-2.14)	(-3.53)	(-1.89)	(-1.56)	(-2.10)	(-4.07)	
Long-term elasticities							
$(QQO_i+QHO_i+QXO_i+SO_i+QIO_i+QGO_i)$ [-1]	1.20**	1.35[(1)] *	1.46*	1.45*	1.38[(2)] *	1.13*	0.87[(3)]*
(PQM_i/PQ_j) [-1]	-0.60	-1.49*	-0.74*	-0.21*	-0.89*	-0.49*	-1.23*
Adjusted R^2	0.55	0.55	0.71	0.89	0.68	0.85	0.71
Durbin Watson	1.36	1.84	1.72	2.13	1.46	2.32	2.03
F-test	6.40	10.01	19.72	60.61	13.43	43.11	14.83

* Parameter from the separately-estimated long-term relation.

** The value of the coefficient was fixed during the estimation.

(1) The estimated long term relation that has been chosen is of the “semi-log” type (on demand). The estimated long-term demand coefficient (which is 3.6e-05) is therefore not an elasticity.

(2) The estimated long term relation that has been chosen is of the “semi-log” type (on demand). The estimated long-term demand coefficient (which is 5.8e-06) is therefore not an elasticity.

(3) The estimated long term relation that has been chosen is of the “semi-log” type (on demand). The estimated long-term demand coefficient (which is 1.2e-05) is therefore not an elasticity.

A = agriculture, B = construction, C = consumer goods, K = equipment goods, L = other market services (than transport and communication), Q = intermediate goods, Z = transport and communication.

The importance of the demand effect turns out to be higher in the import volume equation than in the export volume equation. Indeed, the elasticities of import volumes to demand are either not significantly different from one or even greater than unity in some industries; this is true for the short run and even more for the long run (see table 13). Recall that they were generally smaller than 1 in the case of exports (except for K and Q, see table 10). Above-unity elasticities for imports probably reflect a high import content of production.

The reactivity of import volumes to the relative price term PQM_i/PQ_i seems to be quite different from one branch to another. Indeed, it appears to be relatively low in the short and the long run in the "equipment goods" (K). Conversely, price elasticity would be (relatively) high in the short and the long run in the two other manufacturing industries "consumer goods" (C) and "intermediate goods" (Q). The "construction" industry would be characterized by an insensitiveness of the import volume to price changes in the short run while in the long run, on the contrary, the quantity response would be fairly high. The same pattern, though less marked, would prevail for "transport and communication" (Z) and "agriculture" (A).

The coefficients of the error correction mechanism take values ranging between 0.10 ("equipment goods" (K)) and 0.46 ("intermediate goods" (Q)) in absolute value (see table 13).

c. Prices

Table 14 presents the results of the estimation of import price equations. It shows the estimated parameters – along with their associated Student test statistics –, the adjusted R^2, the Durbin-Watson test statistic and the Fisher test statistic (F-test).

For the sake of comprehensiveness, we note that, again, binary variables proved significant in some equations (branches L and Z).

Table 14: Estimation results of import equations - Prices

IMPORT PRICES		A	B	C	K	L	Q	Z
Constant		0.00	0.00	-0.01	0.00	-0.00	-0.02	0.01
		(0.14)	(0.49)	(-1.57)	(0.09)	(-0.18)	(-3.67)	(0.95)
Short-term elasticities								
	PWM/EXRAT	0.52	0.50*	0.37	0.24	0.23	0.37	0.33
		(4.17)		(3.48)	(3.62)	(5.03)	(4.95)	(3.31)
	PQ_i	0.26	0.00*	0.40	0.40	0.47	0.56	0.34
		(1.68)		(1.67)	(1.61)	(2.34)	(5.66)	(1.36)
ECM								
	PQM_i [-1]	-0.31	-0.10	-0.21	-0.19	-0.19	-0.27	-0.77
		(-2.09)	(-2.89)	(-2.17)	(-4.12)	(-2.45)	(-3.61)	(-4.86)

IMPORT PRICES		A	B	C	K	L	Q	Z
Long-term elasticities								
	PWM/EXRAT [-1]	0.57	1.00*	0.96*	1.00*	0.60*	1.00*	0.63
	PQ_i [-1]**	0.43	0.00	0.04	0.00	0.40	0.00	0.37
Adjusted R^2		0.65	0.39	0.75	0.71	0.79	0.87	0.73
Durbin Watson		1.58	1.60	1.98	1.89	2.49	1.81	1.79
F-Test		13.37	20.18	31.36	24.98	18.95	68.18	16.98

* The value of the parameter was fixed during the estimation.

** For any industry-product, the value of the parameter is equal to 1-coeff (PWM/EXRAT).

A = agriculture, B = construction, C = consumer goods, K = equipment goods, L = other market services (than transport and communication), Q = intermediate goods, Z = transport and communication.

In the short run, Belgian import prices seem to be determined by world prices as well as domestic prices. In the long run, however, world prices seem to play a much larger role.

Indeed, Belgian manufacturing import prices ("consumer goods" (C), "equipment goods" (K) and "intermediate goods" (Q)) are largely, or even fully, determined by world prices[18]; those industries are clearly highly exposed to international competition.

This effect of world prices also dominates – although less markedly – in the branches "transport and communication" (Z), "other market services" (L) and "agriculture" (A).

The dynamic of the error correction mechanism seems to be quite similar in import volume and price equations, with the notable exception of the branch "transport and communication" (Z).

d. Elasticities

Table 15 shows the – aggregated over all industries (except "energy") – reaction of imports following a 1% increase of the total (domestic + foreign) demand addressed to Belgian firms. The sensitiveness of imports to a 1% increase of the ratio (PQM_i/PQ_i) (increased competitiveness of Belgium) is also displayed.

Table 15: Import elasticities

	t	t+3	t+5	Long term
Demand (domestic + foreign)	1.12	1.26	1.31	1.42
Competitiveness	-0.42	-0.52	-0.53	-0.54

The elasticity of imports to total demand exceeds 1, already in the first year (1.12) and more and more as years pass (1.42 after 10 years), probably reflecting a high import content of production.

A 1% improved competitiveness would allow Belgium to decrease its imports by 0.42% in the first year and by a little more than 0.5% afterwards.

[18] Note that for "equipment goods" (K) and "intermediate goods" (Q), the estimated long-run world prices coefficient was initially higher than 1. Nevertheless, we chose to set it to 1.

4. Production block: factors' demand

In this section we describe the determination of sectoral factor demand. For each industry four kinds of production factors are distinguished, i.e. labour, capital, energy and other intermediate inputs. Note that, in HERMES, a distinction has to be made between the approach implemented for the manufacturing and energy branches, on the one hand, and the modelling of other branches (agriculture, construction, transport and communication and other market services), on the other hand. In the first case, a putty-clay technique has been applied, whereas for the other branches, the putty-putty approach has been adopted.

4.1. Factor demand in the manufacturing and energy branches: a putty-clay approach

4.1.1. Theoretical framework

The point of departure for the derivation of factor demand is an ex ante two-level SATO production function[19] with four factors of production: labour, investment, energy and other intermediate consumption. It is assumed that this production function is a twice differentiable, strictly quasi-concave and positive function. It is further assumed that it is separable with respect to the partition of inputs proposed. This implies that the marginal rate of substitution between two inputs which are part of one of the aggregated inputs is independent of the other aggregated inputs.

Two groups of inputs are considered: investment and energy, on the one hand, and labour and other intermediate input, on the other hand. The two inner functions are CES functions for each of the groups of factors considered, whereas the outer function (which links the two groups together) is of the Cobb-Douglas type. Technical progress is assumed to be Hicks-neutral. This gives the following linear homogeneous production function:

$$Q_t^{"} = Ae^{gt}\left[\alpha I_t^{-\rho} + (1-\alpha)E_t^{"-\rho}\right]^{-\gamma/\rho}\left[\beta L_t^{"-\delta} + (1-\beta)M_t^{"-\delta}\right]^{-(1-\gamma)/\delta} \tag{40}$$

Where:

I_t, $E_t^{"}$ and $M_t^{"}$ denote the marginal inputs (resp. gross capital formation, labour, energy and other intermediate inputs) corresponding to the new production capacity;

$Q_t^{"}$ denotes the new production capacity;

$X_{i,t}^{"}$ denotes the marginal input (i=labour, energy, intermediate input) corresponding to the new production capacity;

g is the rate of technical progress;

α is the distribution parameter for the class I-E;

[19] See K. Sato (1967).

β is the distribution parameter for the class L-M;

γ is the share parameter between the two inner classes;

ρ and δ are the CES parameters from which the substitution elasticities can be derived.

This function is of the putty-clay type, i.e. there are substitution possibilities *ex ante*, but not *ex post*. Once a certain amount of capital has been put into production, it will continue to operate during its lifespan in cooperation with constant amounts of the other inputs. The chosen input combination to obtain a gross increment of the production capacity remains unchanged for the whole life time of that particular investment (i.e. the marginal technical coefficients are variable *ex ante*, but fixed *ex post*), so that substitution decisions are only possible at the margin. Input variations are only due to the introduction of new vintages (gross investments) and to the scrapping of unprofitable old vintages. The factor proportions can be derived on the basis of the duality principle: the producer chooses a production technology on the basis of the anticipated factor prices in order to minimize the anticipated cost of the newly installed equipment.

Using a classical cost minimisation procedure, starting from equation (40), one can obtain a demand function for each of the four marginal technical coefficients $K_{i,t}^{"} = X_{i,t}^{"}/Q_t^{"}$ (for i= labour, energy and intermediate input) and $K_t^{"} = I_t/Q_t^{"}$ (for investment).

4.1.2. Estimation results

Next table gives the estimations of the main parameters, obtained from the derived demand functions. Note that ρ and δ are not directly estimated. We estimated, instead, σ=1/(1+ρ) and θ=1/(1+δ).

Table 16: Estimation results of the demand systems

	Intermediate goods (Q)	Equipment goods (K)	Consumer goods (C)	Energy (E)
α	0.99	0.99	0.99	0.99
ß	0.53	0.20	0.11	0.23
σ	0.40	0.48	0.40	0.53
θ	0.70	0.99	0.70	1.12
γ	0.05	0.05	0.06	0.19
g	0.001	0.001	0.004	0.012

Note, for each branch, the high level of α, suggesting a low energy share in the total cost of industries. On the basis of these parameter estimates, it is possible to calculate direct and cross-price elasticities of factor demand (marginal, not total). Table 17 hereafter presents the elasticities computed for the manufacturing and energy branches. The figure at the intersection of line i and column j gives the demand elasticity of factor i to the price of j. Note that these factor substitutions are only possible *ex ante*, because of the putty-clay context.

Table 17: Direct and cross-price elasticities *at the margin*

	Intermediate goods (Q)					Equipment goods (K)			
	I	E	L	M		I	E	L	M
I	-0.76	-0.19	0.47	0.48	I	-0.84	-0.11	0.19	0.76
E	-0.36	-0.59	0.47	0.48	E	-0.36	-0.59	0.19	0.76
L	0.03	0.02	-0.38	0.33	L	0.04	0.01	-0.80	0.75
M	0.03	0.02	0.32	-0.37	M	0.04	0.01	0.19	-0.24
	Consumer goods (C)					Energy (E)			
	I	E	L	M		I	E	L	M
I	-0.69	-0.25	0.17	0.77	I	-0.74	-0.08	0.18	0.63
E	-0.30	-0.64	0.17	0.77	E	-0.20	-0.61	0.18	0.63
L	0.03	0.03	-0.58	0.52	L	0.14	0.05	-0.92	0.73
M	0.03	0.03	0.12	-0.37	M	0.14	0.05	0.21	-0.40

Direct price elasticities are obviously negative. On the basis of the cross price elasticities it can be seen that investments and energy demand are complements: an energy price increase leads, *ceteris paribus*, to a decrease in investments. Labour appears to be a substitute for the other production factors: increasing labour costs leads to a higher demand for the other input categories and *vice versa*.

4.2. Factor demand in the services: a putty-putty approach

4.2.1. Theoretical framework

By analogy with the case of manufacturing branches, the factors' demand for services and construction is obtained by applying a cost-minimization procedure under the constraint of a given production function. The chosen production function takes again the form of a two-level CES nested function, with two composite inputs: K and E on the one hand; L and M on the other hand.

The production process is of the putty-putty type, i.e. there are substitution possibilities *ex ante* as well as *ex post*. That makes the difference with the function estimated for the manufacturing industries, which imposes a putty-clay specification (with substitution possibilities *ex ante* but not *ex post*; see section 4.1 before). Technical progress is assumed to be Hicks-neutral in general.

4.2.2. Estimation results[20]

The following table presents the direct and cross price elasticities obtained (for the long term) for each industry concerned, i.e. six different branches: construction (B), services of credit and insurance institutions (CR), trade, lodging and catering services (HA), rail and road transportation (Z_12), other transport services and communication (Z_56), other market services to households and firms (OS)[21].

Table 18: Direct and cross-price elasticities (long term)

(B)	K	E	L	M	(CR)	K	E	L	M
K	-0.78	0.006	0.20	0.58	K	-1.20	0.00	0.37	0.85
E	0.06	-0.83	0.20	0.58	E	0.11	-1.31	0.37	0.85
L	0.19	0.02	-0.78	0.58	L	0.11	0.00	-0.53	0.43
M	0.19	0.02	0.20	-0.41	M	0.11	0.00	0.19	-0.29
(HA)	**K**	**E**	**L**	**M**	**(OS)**	**K**	**E**	**L**	**M**
K	-0.37	0.05	0.11	0.22	K	-0.51	0.03	0.16	0.33
E	0.66	-0.98	0.11	0.22	E	0.57	-1.05	0.16	0.33
L	0.06	0.00	-0.59	0.52	L	0.16	0.01	-0.65	0.48
M	0.06	0.00	0.26	-0.33	M	0.16	0.01	0.23	-0.41
(Z_12)	**K**	**E**	**L**	**M**	**(Z_56)**	**K**	**E**	**L**	**M**
K	-0.40	0.05	0.30	0.48	K	-0.49	0.01	0.19	0.29
E	0.66	-0.99	0.30	0.48	E	0.57	-1.04	0.19	0.29
L	0.61	0.05	-0.61	0.50	L	0.36	0.01	-0.73	0.37
M	0.61	0.05	0.31	-0.42	M	0.36	0.01	0.23	-0.60

The elasticities are correctly signed, and we can check that energy demand is highly elastic in its price in all branches (exceeding 1 in absolute value in half of the cases). Moreover, cross-elasticities indicate sometimes quite limited substitution possibilities, some being even close to zero. No cross-elasticity exceeds in any case 1. Note also that the complementarity relationship between capital and energy, which is computed in the case of the putty-clay industries, is not verified with the putty-putty industries.

4.2.3. Short term vs long term elasticities

Testing the production block behaviour by means of shocks applied to factors prices illustrates the differences in reactions among industries. An example of these differences can be shown in table 19 which displays the short to long term elasticities of employment.

As can be seen, differences appear between industries, both for the average sensitivity of the labour factor to its cost and the speed of reaction. In general, the short term elasticity is the lowest for the manufacturing sectors, for which a putty-clay technology was selected, while it is relatively high for the construction sector. Long-term elasticities vary between -0.25 (intermediate goods) and -0.82 (construction).

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[20] For a complete presentation of the estimation results, see Bossier (2012).

[21] Note that for the branch Z_34 (air, maritime and fluvial transportation), we adopted an ad hoc methodology.

Table 19: Employment (price) elasticities

	t	t+3	t+5	Long term
Energy (E)	-0.02	-0.15	-0.26	-0.40
Intermediate goods (Q)	-0.03	-0.16	-0.22	-0.25
Equipment goods (K)	-0.02	-0.18	-0.31	-0.48
Consumer goods (C)	-0.03	-0.21	-0.35	-0.48
Construction (B)	-0.65	-0.82	-0.82	-0.82
Services of credit and insurance institutions (CR)	-0.09	-0.18	-0.21	-0.26
Trade, lodging and catering services (HA)	-0.15	-0.37	-0.42	-0.48
Rail and road transportation (Z_12)	-0.22	-0.43	-0.47	-0.55
Other transport services and communication (Z_56)	-0.27	-0.38	-0.42	-0.50
Other market services to households and firms (OS)	-0.47	-0.70	-0.70	-0.70

4.3. Wage and age structure of employment

In this section, we present the submodel that determines employment, as well as wages, by category of worker. For most industries, the wage cost of each labour category is calculated and total employment (computed at the aggregated level of the production function) is allocated across various categories of labour.

4.3.1. Top down allocation and feedback to average labour cost

Within each industry except for agriculture, public administration and voucher-financed household services, total demand for labour and the age and wage structure of salary earning employment are simultaneously determined (see Box 1).

On the one hand, demand for goods and services and the relative cost of labour to capital determine aggregate employment. Labour however is heterogeneous: innate differences in skills and productivity across age and pay categories cause imperfect substitution. Relative wages determine the pay and age structure of total employment. First, employment is allocated across three major wage categories: low-pay jobs (gross quarterly wages up to 5 870 euro in 2005 prices (6 812 euro in 2011) are considered low pay), high-pay jobs and special-employment programmes (employment that qualifies for targeted employers' SSC cuts in programmes involving the long-term unemployed, first-hirings by one-man firms or first-job experiences). Within each of these three categories, employment is allocated between younger workers (aged less than fifty) and older workers (aged fifty or more)[22].

On the other hand, the structure of labour demand affects the average wage cost at each level of allocation, which ultimately feeds back into total demand for labour. Hence, changes in labour-specific

[22] The substitution of employment across age and pay brackets in response to changing factor prices resembles the substitution of labour for capital. It's a gradual process in which new firms or activities with technology, capital labour ratios and employment structures atuned to prevailing factor prices survive whereas existing firms or activities who have failed to adjust to the prevailing market conditions tend to disappear.

wage costs impact on employment through a substitution and volume effect. This matters particularly if selective labour cost reducing policies are simulated[23].

Table 20 gives an overview of the age and pay structure at industry level in 2011. Services of credit and insurance institutions, energy, equipment goods and intermediate goods stand out as high-pay industries, whereas trade, lodging and catering services and rail and road transportation are low-pay activities. The share of older workers – low-pay and high-pay earners together – in the workforce is relatively small in air, maritime and fluvial transportation (of which mostly high-pay earners), trade, lodging and catering services, other market services to households and firms and construction (of which mostly high-pay earners). Relatively large shares of older workers are reported in rail and road transportation, services of credit and insurance institutions and other transport services and communication.

Table 20: Gross wage and age structure of employment at industry level 2011

	Low-wage labour	Of which <50 y.o.	Of which >=50 y.o.	High-wage labour	Of which <50 y.o.	Of which >=50 y.o.	Special-employment
Construction	15.6	14.5	1.1	73.4	58.3	15.1	11.0
Consumer goods	36.7	29.7	6.9	60.2	46.2	13.9	3.2
Services of credit & insur.	9.9	9.0	0.9	88.2	62.9	25.3	1.9
Energy	15.9	13.9	2.0	82.3	61.6	20.7	1.8
Trade, lodging & catering	47.3	41.3	6.1	44.8	34.5	10.3	7.9
Equipment goods	12.3	10.7	1.6	86.2	66.2	20.0	1.5
Other market services	39.9	35.5	4.4	53.5	43.5	10.1	6.5
Intermediate goods	13.2	11.3	1.9	85.0	65.1	19.9	1.8
Health care	30.4	25.3	5.1	64.8	47.8	17.0	4.8
Rail & road transportation	43.3	30.8	12.5	53.0	32.9	20.1	3.7
Air, maritime &fluv.transp.	24.6	23.1	1.5	73.6	59.2	14.3	1.8
Other transp.& communic.	28.6	24.1	4.5	69.9	45.9	24.0	1.6

Gross low-wage ceiling = 6812 euro per quarter; special-employment programmes: employment that qualifies for employers' SSC cuts in the long-term unemployed, first-hirings or first job experience programmes.

23 The way heterogeneity of labour is modeled in HERMES explains the huge impact of labour cost reduction policies targeting low-pay labour on employment in comparison with across-the-board labour cost reduction policies. Indeed, any drop in low-wage rates relative to high-wage rates due to low-pay wage cost reduction policies (be it employers SSC cuts or wage subsidies) skews employment towards low-pay labour, triggering a second-round decrease in the average wage rate on top of the initial average wage cut.

4.3.2. Model

For the sake of convenience, a simplified translog substitution technology, documented in Stockman (2007, paragraph 6), was used. The translog aggregator (41) combines various categories of labour (n_i) into overall employment (n), with category m the numéraire, w_i the wage rate of labour category i, and γ and α_i the translog parameters. The optimal combination is determined by condition (42).

$$\ln n = \sum_{i \neq m} \alpha_i \ln n_i + \left(1 - \sum_{i \neq m} \alpha_i\right) \ln n_m + \left(\frac{\gamma}{2}\right) \sum_i (\ln n_i)^2 - \left(\frac{\gamma}{2}\right) \sum_{i \neq j} \ln n_i \ln n_j \quad (41)$$

$$\frac{w_i n_i}{w_m n_m} = \frac{\alpha_i + \gamma \ln(n_i / \prod_{k \neq i} n_k)}{(1 - \sum_{k \neq m} \alpha_i) + \gamma \ln(n_m / \prod_{k \neq m} n_k)} \quad (42)$$

In the case of HERMES at industry level s, low-pay labour (n_{sLL}), high-pay labour (n_{sHL}) and special employment (n_{sSP}) are combined into total employment (n_s), subject to low-wage labour cost w_{sLL}, high-wage labour cost w_{sHL} and special-employment labour cost w_{sSP}. Condition (43) gives relative demand for high-wage labour; condition (44) gives relative demand for special-employment labour. Given total employment, demand for low-pay labour is obtained residually from identity (45). The parameters to be estimated or calibrated are γ_s, α_{sHL} and α_{sSP}.

$$\frac{w_{sHL} n_{sHL}}{w_{sLL} n_{sLL}} = \frac{\alpha_{sHL} + \gamma_s \ln(n_{sHL} / (n_{sLL} n_{sSP}))}{(1 - \alpha_{sHL} - \alpha_{sSP}) + \gamma_s \ln(n_{sLL} / (n_{sHL} n_{sSP}))} \quad (43)$$

$$\frac{w_{sSP} n_{sHL}}{w_{sLL} n_{sLL}} = \frac{\alpha_{sSP} + \gamma_s \ln(n_{sSP} / (n_{sLL} n_{sHL}))}{(1 - \alpha_{sHL} - \alpha_{sSP}) + \gamma_s \ln(n_{sLL} / (n_{sHL} n_{sSP}))} \quad (44)$$

$$n_s = n_{sLL} + n_{sHL} + n_{sSP} \quad (45)$$

Similarly, demand for young and older workers within each wage category can be derived. Given the cost of young high-wage (w_{sINJHL}) and older high-wage labour (w_{sOWHL}), young high-wage employment (n_{sINJHL}) and older high-wage employment (n_{sOWHL}) are determined by condition (46) and identity (47), with γ_{sHL} and α_{sOWHL} the high-wage translog parameters.

$$\frac{w_{sOWHL} n_{sOWHL}}{w_{sNJHL} n_{sNJHL}} = \frac{\alpha_{sOWHL} + \gamma_{sHL} \ln(n_{sOWHL} / n_{sNJHL})}{(1 - \alpha_{sOWHL}) + \gamma_{sHL} \ln(n_{sNJHL} / n_{sOWHL})} \quad (46)$$

$$n_{sHL} = n_{sNJHL} + n_{sOWHL} \quad (47)$$

Applying the same methodology, condition (48) and identity (49) determine young low-wage employment (n_{sINJLL}) and older low-wage employment (n_{sOWLL}), given the cost of young low-wage (w_{sINJLL}) and older low-wage labour (w_{sOWLL}) and the low-wage translog parameters γ_{sLL} and α_{sOWLL}

$$\frac{w_{sOWLL} n_{sOWLL}}{w_{sNJLL} n_{sNJLL}} = \frac{\alpha_{sOWLL} + \gamma_{sLL} \ln(n_{sOWLL} / n_{sNJLL})}{(1 - \alpha_{sOWLL}) + \gamma_{sLL} \ln(n_{sNJLL} / n_{sOWLL})} \quad (48)$$

$$n_{sLL} = n_{sNJLL} + n_{sOWLL} \quad (49)$$

4.3.3. Calibration results

Wage rates and employment in (43)-(49) are full-time equivalents. Short of long time series, the substitution technology in the labour market is quantified by calibrating the optimality conditions on the most recent observations (2011 in this vintage) and not by time-series based econometrics, which would be first-best. The compensated elasticities[24] and the Allen substitution elasticities[25] are reported in table 21 (wage structure), table 22 (age structure within low-paid employment) and table 23 (age structure within high-paid employment). Table 24 reports proxies for the elasticities of substitution between pairs of wage categories[26].

From the compensated elasticities, it follows that low-pay and high-pay labour are invariably substitutes. On the whole special-employment jobs are substitutes to low-pay and high-pay labour as well, except in some industries where special-employment jobs and low-pay labour are complements. The (proxies for the) elasticities of substitution are near unity in the case of low-pay and high-pay labour, suggesting that labour cost reducing policies, whether selective or across-the-board, would leave the budget shares of the total labour cost largely unaffected.

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24 For given total employment, compensated elasticities quantify the substitution effect of changes in the wage costs of the various categories of labour (young or older, low-pay, high-pay or special employment) on demand for a particular segment of labour (say high-wage labour). Positive-signed cross compensated elasticities identify pairs of substitutes, negative-sign compensated elasticities reveal pairs of complements. For the allocation to be dynamically stable, the absolute value of the own compensated elasticities are not to be smaller than the cross compensated elasticities. Theoretically, compensated elasticities should sum to zero for each labour category, meaning that - for given total employment - demand for a particular labour category (say high-wage labour) should not change if all wage rates change in the same proportions. The adding-up-tests in Table 21 to Table 23 verify whether the data corroborate the theoretical adding-up constraint on the compensated elasticities.

25 The Allen elasticities of substitution are equal to the compensated elasticities, divided by the share in total labour cost of the labour category of which the wage cost has changed. The Allen elasticities of substitution are supposed to be symmetrical: e.g. the impact of a change in low-wage costs on demand for high-wage labour should equal the impact of a change in high-wage costs on demand for low-wage labour. Whether the symmetry constraint holds can be verified from the off-diagonal elements in the Allen elasticities of substitution matrix.

26 Direct elasticities of substitution measure the percentage impact of a percentage change in relative factor costs on factor ratios.

Table 21: Compensated elasticities and Allen-substitution elasticities of low-pay, high-pay and special-programme employment
Calculated from calibrated translog parameters (2011)

Industry	Labour	Compensated elasticities w.r.t. the wage rate of				Allen-substitution elasticities w.r.t. the wage rate of		
		Low-pay	High-pay	Special jobs	Test adding-up	Low-pay	High-pay	Special jobs
Construction	Low-pay	-0.69	0.65	0.02	-0.02	-5.27	0.83	0.23
	High-pay	0.13	-0.24	0.09	-0.01	1.03	-0.31	1.00
	Special	0.03	0.63	-0.68	-0.02	0.19	0.82	-7.40
Consumer goods	Low-pay	-0.56	0.53	0.00	-0.02	-2.00	0.77	0.15
	High-pay	0.31	-0.35	0.02	-0.02	1.13	-0.51	0.90
	Special	0.05	0.38	-0.46	-0.03	0.18	0.54	-19.49
Serv. of credit & insurance	Low-pay	-0.76	0.81	-0.06	-0.01	-12.39	0.88	-4.00
	High-pay	0.08	-0.11	0.02	-0.01	1.33	-0.11	1.15
	Special	-0.25	0.78	-0.54	-0.02	-4.11	0.84	-37.96
Energy	Low-pay	-0.70	0.72	-0.04	-0.01	-7.12	0.81	-3.37
	High-pay	0.13	-0.15	0.01	-0.01	1.29	-0.17	1.15
	Special	-0.34	0.64	-0.32	-0.03	-3.50	0.72	-27.58
Trade, lodging & catering	Low-pay	-0.47	0.41	0.04	-0.02	-1.31	0.69	0.84
	High-pay	0.41	-0.49	0.06	-0.02	1.16	-0.83	1.08
	Special	0.30	0.30	-0.63	-0.03	0.84	0.50	-11.70
Equipment goods	Low-pay	-0.76	0.78	-0.04	-0.01	-8.40	0.87	-2.96
	High-pay	0.11	-0.13	0.01	-0.01	1.17	-0.14	1.03
	Special	-0.29	0.73	-0.46	-0.02	-3.25	0.81	-37.68
Other market services	Low-pay	-0.54	0.49	0.03	-0.02	-1.96	0.72	0.64
	High-pay	0.34	-0.41	0.05	-0.02	1.26	-0.61	1.09
	Special	0.21	0.40	-0.63	-0.03	0.77	0.58	-13.60
Intermediate goods	Low-pay	-0.76	0.78	-0.03	-0.01	-8.82	0.87	-2.45
	High-pay	0.11	-0.14	0.01	-0.01	1.32	-0.15	1.18
	Special	-0.24	0.73	-0.51	-0.02	-2.75	0.81	-41.29
Health care	Low-pay	-0.62	0.59	0.01	-0.02	-2.75	0.79	0.29
	High-pay	0.26	-0.31	0.03	-0.02	1.17	-0.42	1.16
	Special	0.04	0.49	-0.56	-0.03	0.17	0.66	-18.73
Rail & road transportation	Low-pay	-0.49	0.46	0.01	-0.02	-1.23	0.81	0.30
	High-pay	0.37	-0.41	0.02	-0.02	0.92	-0.72	0.60
	Special	0.13	0.27	-0.43	-0.03	0.32	0.48	-12.37
Air,maritime & fluvial transp.	Low-pay	-0.47	0.49	-0.03	-0.02	-3.61	0.57	-2.39
	High-pay	0.15	-0.17	0.01	0.00	1.17	-0.20	1.01
	Special	-0.25	0.37	-0.14	-0.02	-1.92	0.43	-10.11
Other transp. & communication	Low-pay	-0.62	0.61	-0.01	-0.02	-2.78	0.80	-0.69
	High-pay	0.25	-0.28	0.01	-0.02	1.12	-0.36	0.85
	Special	-0.13	0.45	-0.36	-0.04	-0.58	0.59	-25.52

Exogenous wage and age structure in the case of agriculture, voucher-financed household services.

Table 22: Compensated elasticities and Allen-substitution elasticities of younger (up to 50 years old) and older workers (at least 50 years old) within low-pay employment
Calculated from calibrated translog parameters (2011)

Industry	Employment	Compensated elasticities w.r.t. the wage rate of			Allen-substitution elasticities w.r.t. the wage rate of	
		Low-pay & <50 y.o.	Low-pay & >=50 y.o.	Test adding-up	Low-pay & <50 y.o.	Low-pay & >=50 y.o.
Construction	Low-pay & <50 y.o.	-0.03	0.03	0.00	-0.03	0.42
	Low-pay & >=50 y.o.	0.40	-0.41	-0.01	0.43	-5.49
Consumer goods	Low-pay & <50 y.o.	-0.04	0.05	0.00	-0.05	0.25
	Low-pay & >=50 y.o.	0.19	-0.20	-0.01	0.23	-1.06
Serv. of credit & insurance	Low-pay & <50 y.o.	-0.04	0.04	0.00	-0.04	0.41
	Low-pay & >=50 y.o.	0.35	-0.36	-0.01	0.38	-4.10
Energy	Low-pay & <50 y.o.	-0.04	0.04	0.00	-0.05	0.35
	Low-pay & >=50 y.o.	0.29	-0.30	-0.01	0.33	-2.45
Trade, lodging & catering	Low-pay & <50 y.o.	-0.04	0.04	0.00	-0.05	0.33
	Low-pay & >=50 y.o.	0.29	-0.30	-0.01	0.33	-2.27
Equipment goods	Low-pay & <50 y.o.	-0.04	0.04	0.00	-0.05	0.34
	Low-pay & >=50 y.o.	0.29	-0.30	-0.01	0.33	-2.33
Other market services	Low-pay & <50 y.o.	-0.04	0.04	0.00	-0.05	0.37
	Low-pay & >=50 y.o.	0.32	-0.33	-0.01	0.36	-3.00
Intermediate goods	Low-pay & <50 y.o.	-0.04	0.04	0.00	-0.05	0.31
	Low-pay & >=50 y.o.	0.26	-0.27	-0.01	0.30	-1.86
Health care	Low-pay & <50 y.o.	-0.04	0.05	0.00	-0.05	0.28
	Low-pay & >=50 y.o.	0.22	-0.23	-0.01	0.26	-1.42

Exogenous age structure in the case of rail & road transportation, air, maritime & fluvial transportation and other transport services & communication.

Table 23: Compensated elasticities and Allen-substitution elasticities of younger (up to 50 years old) and older workers (at least 50 years old) within high-pay employment
Calculated from calibrated translog parameters (2011)

Industry	Employment	Compensated elasticities w.r.t. the wage rate of			Allen-substitution elasticities w.r.t. the wage rate of	
		High-pay & <50 y.o.	High-pay & >=50 y.o.	Test adding-up	High-pay & <50 y.o.	High-pay & >=50 y.o.
Construction	High-pay & <50 y.o.	-0.04	0.05	0.00	-0.06	0.21
	High-pay & >=50 y.o.	0.17	-0.18	-0.01	0.22	-0.80
Consumer goods	High-pay & <50 y.o.	-0.04	0.04	0.00	-0.06	0.17
	High-pay & >=50 y.o.	0.14	-0.14	-0.01	0.18	-0.58
Serv. of credit & insurance	High-pay & <50 y.o.	-0.03	0.03	0.00	-0.05	0.10
	High-pay & >=50 y.o.	0.08	-0.09	0.00	0.13	-0.26
Energy	High-pay & <50 y.o.	-0.04	0.04	0.00	-0.06	0.13
	High-pay & >=50 y.o.	0.12	-0.12	0.00	0.17	-0.39
Trade, lodging & catering	High-pay & <50 y.o.	-0.04	0.04	0.00	-0.06	0.17
	High-pay & >=50 y.o.	0.14	-0.15	0.00	0.19	-0.58
Equipment goods	High-pay & <50 y.o.	-0.04	0.04	0.00	-0.06	0.17
	High-pay & >=50 y.o.	0.14	-0.14	0.00	0.19	-0.57
Other market services	High-pay & <50 y.o.	-0.05	0.05	0.00	-0.06	0.21
	High-pay & >=50 y.o.	0.20	-0.20	-0.01	0.25	-0.92
Intermediate goods	High-pay & <50 y.o.	-0.04	0.04	0.00	-0.06	0.16
	High-pay & >=50 y.o.	0.14	-0.14	0.00	0.18	-0.54
Health care	High-pay & <50 y.o.	-0.04	0.04	0.00	-0.05	0.13
	High-pay & >=50 y.o.	0.11	-0.11	0.00	0.15	-0.37

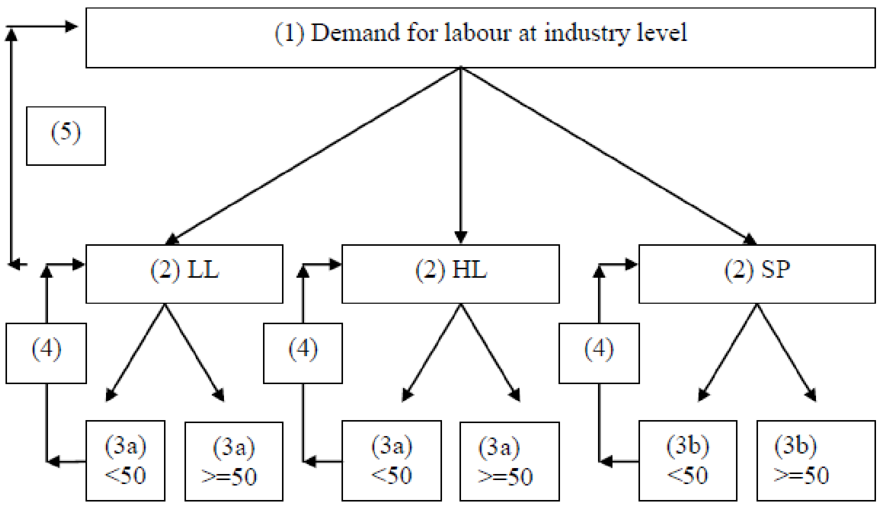
Exogenous age structure in the case of rail & road transportation, air, maritime & fluvial transportation and other transport services & communication.

Table 24: *Ex post* elasticities of employment ratios (n_i/n_j) to relative labour costs (w_i/w_j)
Calculated from calibrated translog parameters (2011) and a 10% shock in relative labour costs

	i=HL j=LL	i=SP j=LL	i=LL j=HL	i=SP j=HL	i=HL j=SP	i=LL j=SP
Construction	-0.94	-0.81	-1.02	-1.00	-0.88	-0.80
Consumer goods	-1.00	-0.69	-1.02	-0.83	-0.54	-0.52
Services of credit & insurance	-0.97	-0.58	-1.05	-1.01	-0.63	-0.55
Energy	-0.94	-0.40	-1.00	-0.89	-0.38	-0.32
Trade, lodging & catering	-1.02	-0.88	-1.03	-0.90	-0.78	-0.76
Equipment goods	-0.99	-0.52	-1.05	-0.98	-0.53	-0.47
Other market services	-1.01	-0.85	-1.04	-0.93	-0.78	-0.75
Intermediate goods	-1.01	-0.59	-1.06	-0.99	-0.60	-0.54
Health care	-1.01	-0.75	-1.04	-0.91	-0.67	-0.64
Rail & road transportation	-0.99	-0.70	-1.00	-0.77	-0.51	-0.50
Air,maritime & fluvial transp.	-0.71	-0.25	-0.75	-0.60	-0.17	-0.11
Other transp.& communication	-1.00	-0.55	-1.02	-0.84	-0.42	-0.39

HL = High-pay employment, LL = Low-pay employment, SP = special employment programmes.

Box 1 The structure of employment and average cost of labour at industry level



1. The average cost of labour relative to other factor prices determines demand for labour.
2. Low-wage, high-wage and special-employment cost of labour determine the allocation of total employment across LL (low-pay earners), HL (high-pay earners) and SP (special-employment programmes).
3. (3a) The age-specific costs of labour determine the allocation between younger and older employment within each wage category (be it LL or HL) ; (3b) SP: Exogenous age structure.
4. The age structure feeds back into the average cost of labour of LL, HL and SP.
5. The wage structure feeds back into the industry's overall average cost of labour .

4.3.4. Behavioural equations and accounting identities

Aside from the behavioural equations and identities (43)-(49), the employment allocation submodel consists of the accounting identities (50)-(58).

a. Older low-pay (OWLL), young low-pay (NJLL), older high-pay (OWHL), young high-pay (NJHL) and special-programme employment (SP)

Be wg_i the gross wage, t_{si} the employers' SSC rate and s_{si} the wage subsidy rate, then the wage cost (w_{si}) of employment category i (i = OWLL, NJLL, OWHL, NJHL, SP), to be used in (44), (46) and (48)[27], is defined by (50).

$$w_{si} = wg_{si}(1 + t_{si} - s_{si}); \tag{50}$$

The SSC rate is the sum of the legal Social Security rate (exogenous, net of SSC reductions[28]), the imputed SSC rate i.e. the rate which redundancy payments and early-retirement payments are imputed to

27 Some inertia is built into the model by using 5-year moving averages of w_{si} (i = OWLL, OWHL, NJLL, NJHL, SP, LL, HL) in (43), (44), (46) and (48).

28 The SSC cuts are forecast bottom-up, starting from a highly disaggregated dataset (courtesy of the National Social Security Office - RSZ/ONSS).

the wage bill (endogenous[29]), the corporate private pension contribution rate (exogenous) and the corporate private health care contribution (endogenous[30]).

The wage subsidy rate (by and large exogenous) covers a broad range of wage bill reducing measures financed by various government entities (Social Security, federal government, regional governments). Some subsidies are across the board, some aimed at specific industries (health care, voucher-financed household services, merchant navy, tugging, dredging, professional sports), particular categories of employees (long-term unemployed, researchers) or worked hours (overtime and shift-organised employment).

The gross wage rate of young, high-pay labour (wg_{sNJHL}), whether exogenous or endogneous, is the reference rate to which the gross wage rates of the other labour categories are aligned, bar a premium or discount ($disc_{sy}$):

$$\Delta \ln wg_{sy} = disc_{sy} + \Delta \ln wg_{sNJHL} \qquad \text{with y = OWLL, OWHL, NJLL, SP} \tag{51}$$

The wage bill (WB_{si}), the gross wage bill ($WBFU_{si}$), the SSCs (SSF_{si}) and wage subsidies ($LSUB_{si}$) for i = OWLL, NJLL, OWHL, NJHL, or SP are defined by (52).

$$SSF_{si} = t_i WBFU_{si};\ LSUB_{si} = s_i WBFU_{si};\ WBFU_{si} = wg_{si} n_{si};\ WB_{si} = WBFU_{si} + SSF_{si} - LSUB_{si} \tag{52}$$

b. Low-pay (LL) and high-pay employment (HL)

The wage costs of low-pay and high-pay employment (w_{sl}, l=LL, HL), to be used in (43) and (44), are defined by (53):

$$w_{sl} = \frac{n_{sOWl} w_{sOWl} + n_{sNJl} w_{sNJl}}{n_{sOWl} + n_{sNJl}}; \tag{53}$$

c. Total at industry level

Summation over OWLL, NJLL, OWHL, NJHL, and SP gives the industry totals for the wage bill (WB), the gross wage bill (WBFU), the SSCs (SSF) and the wage subsidies (LSUB), as defined by (54)-(57):

$$WB_s = WB_{sOWLL} + WB_{sNJLL} + WB_{sOWHL} + WB_{sNJHL} + WB_{sSP} \tag{54}$$

$$WBFU_s = WBFU_{sOWLL} + WBFU_{sNJLL} + WBFU_{sOWHL} + WBFU_{sNJHL} + WBFU_{sSP} \tag{55}$$

$$SSF_s = SSF_{sOWLL} + SSF_{sNJLL} + SSF_{sOWHL} + SSF_{sNJHL} + SSF_{sSP} \tag{56}$$

$$LSUB_s = SSF_{sOWLL} + LSUB_{sNJLL} + LSUB_{sOWHL} + LSUB_{sNJHL} + LSUB_{SsP} \tag{57}$$

Equation (58) defines the industry's average gross wage wg_s.

[29] Linked to forecasts of the number of early retired.
[30] Linked to future employment and the health care price index.

$$wg_s = \frac{WBFU_{sOWLL}+WBFU_{sNJLL}+WBFU_{sOWHL}+WBFU_{sNJHL}+WBFU_{sSP}}{n_s} \tag{58}$$

4.3.5. Gross wages

a. Gross wages: exogenous or endogenous?

Gross wages are either exogenous or endogenous. If exogenous, gross wages may obey the outcome of the 2-year wage bargaining agreements or the wage growth determined by official wage policies. If such benchmarks are not available, an out-of-model econometric labour cost equation that links the macroeconomic labour cost to the unemployment rate, average labour productivity and the tax wedge supplies an estimate of future gross wage increases. This estimate is then applied to the gross wages of all labour categories within each industry.

In the exogenous wage version of HERMES, the employment allocation submodel consists of (43)-(49) and (50)-(58).

The endogenous wage version links the gross wages at the industry level to the health index inflation, the industry's value added deflator, the unemployment rate, macroeconomic labour productivity and industry-specific labour productivity.

b. The econometrics of endogenous wages

The econometrics rely on LSQ estimates of average gross wage equations at industry level (average in the sense that employment is not disaggregated), with subscript 's' referring to industry s, 'p_h'the health index, 'p_s' the sectoral output price index, 'wg_s' the industry's average gross wage rate in industry s, 'n_s' employment and 'y_s' sectoral output, 'ur' the unemployment rate (FPB definition), 'n_b' total employment in all businesses, 'y_b' total output of all businesses (59). Implicitly, any industry's average nominal gross wage rate's optimal rate of growth is modeled by (60), whereas (59) (or (61)) is a partial adjustment mechanism. The equilibrium growth rate of gross wages rises above autonomous wage growth (w_{s1}) if productivity exceeds the producer real gross wage (w_{s4}>0) or because of inflation (w_{s2}>0). A rise in the unemployment rate lowers the equilibrium growth rate of gross wages (Phillips curve effect, w_{s3}<0). The coefficients of interest, i.e. the ones measuring economic feedback, are reported in table 25.

$$\begin{aligned}\triangle\,(\triangle \ln wg_s) = w_{s10}\Bigg(& w_{s1} + w_{s2} \triangle \ln p_h + w_{s3} ur + w_{s4}\left(ws_5 \ln\left(\frac{y_s}{n_s}\right) + (1 - w_{s5}) \ln\left(\frac{y_b}{n_b}\right) - \right.\\ & \left.\ln\left(\frac{wg_s}{p_s}\right)_{-1}\right) - (\triangle \ln wg_s)_{-1}\Bigg)\end{aligned} \tag{59}$$

$$\overline{\triangle \ln wg_s} = w_{s1} + w_{s2} \triangle \ln p_h + w_{s3} ur + w_{s4}\left(w_{s5} \ln\left(\frac{y_s}{n_s}\right) + (1 - w_{s5}) \ln\left(\frac{y_b}{n_b}\right) - \ln\left(\frac{wg_s}{p_s}\right)_{-1}\right) \tag{60}$$

$$\triangle \ln wg_s - (\triangle \ln wg_s)_{-1} = w_{s10}\left(\overline{\triangle \ln wg_s} - (\triangle \ln wg_s)_{-1}\right) \tag{61}$$

Table 25: Gross wage equations
Based on LSQ estimates of (59), 1982-2011

	A	B	C	CR	E	HA	K	OS	Q	SA	Z_12	Z_34	Z_56
(w_{10})	0.99	0.99	0.84	0.84	0.98	0.54	0.99	0.93	0.95	0.99	0.95	0.99	0.95
(w_3)	-0.20	-0.17	-0.44	-0.41	-0.56	-0.60	-0.34	-0.25	-0.20	-0.19	-0.62	-0.26	-0.10
(w_4)	0.08	0.18	0.20	0.09	0.22	0.07	0.27	0.22	0.51	0.08	0.19	0.06	0.36
(w_5)	0.95	0.75	0.36	0.12	0.69	0.90	0.57	0.50	0.73	0.45	0.95	0.94	0.95
DW	2.30	2.15	2.09	1.76	2.02	2.05	2.31	2.16	1.82	2.45	2.08	2.51	1.65
Adj R^2	0.60	0.64	0.54	0.59	0.54	0.56	0.59	0.58	0.52	0.62	0.60	0.58	0.59

A = agriculture, B = construction, C = consumer goods, CR = Services of credit & insurance institutions, E = energy, HA = Trade, lodging & catering services, K = equipment goods, OS = other market services to households & firms, Q = intermediate goods, SA = health care, Z_12 = rail & road transportation, Z_34 = air, maritime & fluvial transportation, Z_56 = other transport services & communication.
w_{10} = Partial adjustment
w_3 = Phillips curve effect (Unemployment rate)
w_4 = Labour productivity effect
w_5 = Share of sector productivity in productivity effect
The health index parameters are constrained to be equal to one.

c. Endogenous wages in HERMES

As such, except for agriculture for which the segmentation of employment is not modelled, equation (59) does not figure in HERMES. Instead, the flexible wage format is imposed on the gross wages of young high-wage labour. Any discrepancy at industry level between average equilibrium wage growth and lagged observed average wage growth (wg_s) is fed into the gross wage growth of young high-wage labour (wg_{sNJHL}) through (62) and passed on to the gross wages of the other employment categories (NJLL, OWLL, OWHL, SP) through (51).

$$\begin{aligned}\Delta\left(\Delta \ln wg_{sNJHL}\right) = w_{s10}\Bigg(w_{s1} + w_{s2}\,\Delta \ln p_h + w_{s3}ur + w_{s4}\Bigg(w_{s5}\ln\left(\frac{y_s}{n_s}\right) + \\ (1-w_{s5})\ln\left(\frac{y_b}{n_b}\right) - \ln\left(\frac{wg_s}{p_s}\right)_{-1}\Bigg) - \left(\Delta \ln wg_{sNJHL}\right)_{-1}\Bigg)\end{aligned} \tag{62}$$

In the endogenous wage version of HERMES, the employment allocation submodel, (43)-(49) and (50)-(58), is supplemented with equation (62) for all industries except for agriculture and with equation (59) for agriculture.

5. Energy and greenhouse gases (GHG) emissions

Energy products play a dual role in the HERMES model: as a production factor in the production function for each branch and as a production branch itself. For each of the eight energy products (see table 1), demand, prices and an input-output equilibrium, in quantity and in value terms, are calculated. The latter leads to a central application of the energy module in the model: the compilation of annual physical energy balance sheets according to the EUROSTAT format[31], from which, in turn, can be derived CO_2 emission (from energy use) sheets.

5.1. Energy demand by branches and households

5.1.1. Branches

Demand per product i of branch s is computed from total energy demand, itself calculated in the production block for each branch, together with the other production factors (see chapter 4).

$$EO_sE_i = ME_{i,s} \,.\, QEU_s / PE_{i,s} \tag{63}$$

Where:

$ME_{i,s}$ is the share of product i into total energy cost of branch s (for s=1,...,15);

$PE_{i,s}$ is the price of energy product i for branch s and QEU_s the total energy demand (or energy cost) of branch s in current prices.

$$QEU_s = QEO_S \,.\, PE_S \tag{64}$$

Where:

QEO_s, the total energy demand in constant prices, is computed in the production block;

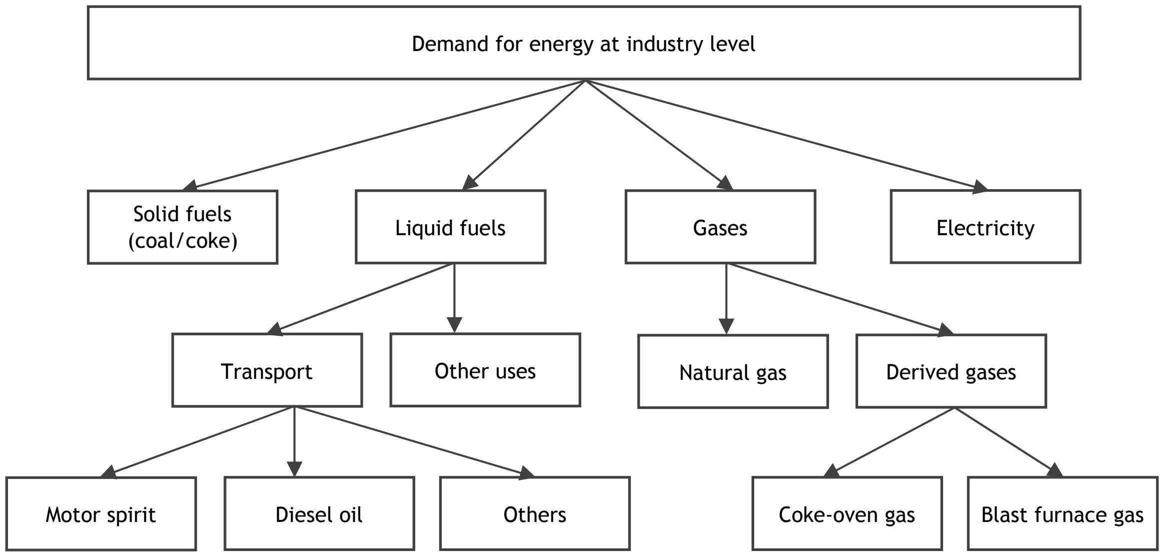
PE_s, the average price of energy for branch s is presented below.

Note that $ME_{i,s}$, the share of each energy product in the total energy cost of branch s, is computed via the use of an allocation model (see point 5.5).

One special attention is given to the treatment of petroleum products. For each sector, petroleum products are distinguished according to their use: transportation or heating purposes. Moreover, fuels used for transportation are divided between the main fuels used on the Belgian territory (gasoline, diesel and jet fuel if pertinent). The following graph gives an overview of the computation of energy demand at each modelling stage.

31 See for instance: EUROSTAT (2013).

Box 2 The structure of energy demand in HERMES by industry



5.1.2. Households

Energy demand of households per product is computed from the consumption allocation module (see point 2.2). This allocation module distinguishes, in a first stage, the total energy demand of the households (group 3), in a second stage the fuel consumption for heating, power and fuels for transportation (the variable $ECOE_i$) and, in a final stage, the different products used (solid, liquid and gas for heating purposes; electricity for power; gasoline and diesel for transportation).

5.2. Transformation activities

An important part of energy is not used directly for heating, power and transportation purposes but is transformed in other energy forms. A special module computes the inputs used to produce electricity (solid fuels, liquid fuels, gases, renewables, nuclear fuels and others) as well as fuels transformed by refineries, coke-oven plants, blast-furnace plants and district heating plants.

As far as power plants are considered, HERMES takes as exogenous the structure of the electricity production (% of nuclear plants, % of thermal power plants working with coal, liquid fuel, gas, % of renewables,...)[32]. Starting from this structure, the energy module of HERMES computes the inputs required for each type of power plant.

$$EELE_i = EQDJ_i \; / \; EFF_i \tag{65}$$

[32] For forecasting purposes, the evolution of electricity production structure comes from other studies published by the FPB. See for instance: Devogelaer and Gusbin (2007).

Where:

$EELE_i$ is the quantity of product i used as input for electricity production (in Terajoules-Tj);

$EQDJ_i$ is the production of electricity (in Tj) issued from power plants using energy product i;

EFF_i is an efficiency coefficient[33] for the conversion of product i into electricity.

In 2011, the following structure was observed for Belgium.

Table 26: Electricity generation by fuel in Belgium - 2011
(in % of total gross electricity production)

	% of total gross electricity generation
Nuclear	54.2
Thermal	35.0
- Solid	3.8
- Liquid	0.3
- Gas (natural and derived gas)	30.8
Renewables	9.2
- Hydro-wind	2.8
- Biomass and waste	5.1
- Solar, tidal	1.3
Other fuels	1.6
TOTAL	100.0

5.3. Computation of total energy needs and of gross inland consumption

5.3.1. Total energy needs per energy product

Once energy demands per branch and agent and transformation activities have been computed, the model proceeds to the calculation of total energy needs per product.

$$\begin{aligned} FDE_i = EO_sE_i.\alpha_{i,s} + ECOE_i.\beta_i + EOJE_i + EELE_i + EO_jE_i + ESOE_i + ESTE_i \\ + EPTE_i + EMPE_i + EXOE_i.\gamma_i \end{aligned} \quad (66)$$

Where:

FDE_i is the total demand of product i (in Tj);

EO_sE_i is the consumption of product i by branch s (in constant prices - see point 5.1);

$ECOE_i$ is the consumption of product i by private consumption (in constant prices - see point 5.1);

$EOJE_i$ is the consumption of product i by the energy branch (in Tj);

$EELE_i$ is the transformation of product i into electricity (in Tj - see point 5.2);

EO_jE_i is the transformation of product i into product j (other than electricity);

[33] The efficiency coefficient (for nuclear and thermal power plants) varies from about 33% (nuclear power plants) to more than 50% (for CCGT power plants).

$ESOE_i$ is the stock change (in Tj);

$ESTE_i$ is the consumption of bunkers (only for petroleum products; in Tj);

$EPTE_i$ is the distribution losses (only for electricity; in Tj);

$EMPE_i$ is the consumption of product i for non energy needs (in Tj);

$EXOE_i$ is the export of product i (in constant prices);

α_i, β_i and γ_i are conversion coefficients (from constant prices to Tj).

Note that EO_SE_i, $ECOE_i$ and $EXOE_i$ are expressed in billions of constant euros and must be converted into Terajoules (by means of conversion coefficients) to compute FDE_i.

5.3.2. Computation of gross inland consumption

Finally the (per energy form and total) gross inland consumption can be computed.

$$CABE_i = FDE_i - EXOE_i - ESOE_i - ESTE_i - EXOE_i - EQDE_i \quad (67)$$

Where:

$CABE_i$ is the gross inland consumption of energy product i;

$EQDE_i$ is the transformation output of energy product i (i= coke, petroleum products, derived gas, renewables, derived heat and electricity).

The equation (67) can also be written as:

$$CABE_i = EQE_i + EMOE_i.\alpha_i + ESOE_i - EXOE_i.\beta_i \quad (68)$$

Where:

EQE_i is the primary production of product i[34];

$EMOE_i$ is the import of product i;

α and β are conversion factors (from constant euros to Tj).

Finally, the model computes the total gross inland consumption as the sum of the gross inland consumption of each energy product.

$$CABE_{total} = \sum_i CABE_i \quad (i = 1,8) \quad (69)$$

[34] The primary production (and recovered products) concerns the following products in Belgium: hard coal, feedstocks, nuclear heat, renewables and other products.

5.4. Other relations

5.4.1. Transformation output per energy product

a. Electricity

Total electricity production is explained by the evolution of total electricity needs and by a relative price comparing the production costs of electricity in Belgium to the price of total electricity resources.

$$EPSE_i = f\ (FDE_i, PEFE_i\ /\ PFME_i) \tag{70}$$

Where:

$EPSE_i$[35] is the total electricity production (expressed in Tj or GWh);

FDE_i is the total demand for electricity (see point 5.3.1);

$PEFE_i$ is the production cost of electricity in Belgium;

$PFME_i$ is the price of total electricity resources.

b. Other products

Like in the case of electricity, the production of other products is explained by a relation including the volume of energy needs (for the product concerned) and a relative price. This is especially the case for refineries and coke-oven plants. The computation of derived gas production is a bit different. The production of coke-oven gas depends on coke production, when blast-furnace gas production depends on iron and steel activity in the country (and on the use of coke by this industry).

5.4.2. External trade

a. Exports

Exports are explained by the level of national production and by the export price divided by the absorption price (as a proxy for the profitability of foreign markets relative to the domestic market).

$$EXOE_i = f\ (EPSE_i, PEXE_i/PAE_i) \tag{71}$$

Where:

$EXOE_i$ is the exports of energy product i (in constant euros);

$PEXE_i$ is the export price of energy product i (equal to the “world “price of energy product i);

PAE_i is the absorption price of energy product i[36].

[35] For the allocation of total electricity production between the different forms of power plants, see point 5.2.

[36] The absorption price is itself a weighted average of internal product price and net import price.

b. Imports

Imports make sure that, for each energy product, uses and resources are equal to each other. They are obtained as the difference between the uses and the domestic resources to cover those uses. For products which are not imported, such as manufactured gas, imports will simply be zero as domestic production covers the uses by definition, while all uses will be imported if there is no domestic production at all (e.g. for crude oil).

5.5. Allocation model for the energy products used by branches

The analysis of the substitution behaviour between energy products takes a central place in the model. We saw (see chapter 4) that the total energy demand of each industry is computed with demand functions derived from two-level CES nested functions, in which energy is supposed to be weakly separable from the other inputs. At a second stage, expenditure on energy is broken down into expenditures in the different energy products (see point 5.1.1). To this end, we used (as in the case of the consumption allocation module) the Almost Ideal Demand System (AIDS) which can be basically written (we omit the index s for industries):

$$ME_i = \alpha_i + \sum_i \gamma_{ij} \ln PE_j + \beta_i \ln \frac{X}{PE} \qquad i,j = 1, \dots ,8 \tag{72}$$

Where:

$$\ln PE = \alpha_0 + \sum_i \alpha_i \ln PE_i + 0.5 \sum_i \sum_j \gamma_{ij} \ln PE_i \ln PE_j \tag{73}$$

ME_i is the budget share of energy product i;

PE_i is the price of ith energy product;

PE is the price per unit of energy;

X is the nominal income (here the total energy demand of the branch).

The usual restrictions (adding-up, homogeneity and symmetry) are applied to this model (see point 2.2.1 for a detailed presentation of the model):

$$\sum_i \alpha_i = 1 \quad \sum_i \beta_i = 0 \quad \sum_i \gamma_{ij} = 0 \tag{74}$$

$$\sum_j \gamma_{ij} = 0 \tag{75}$$

$$\gamma_{ij} = \gamma_{ji} \tag{76}$$

Uncompensated price and income elasticities can be derived (see point 2.2.1 for a detailed presentation).

The possible deviation of short term demand from the long term target suggests to introduce a partial adjustment process of the form:

$$\vartriangle ME_i = \lambda\,(ME^*_{i,t} - ME_{i,t-1}) \tag{77}$$

Where:

ME^*_i is the optimal level of the ith product share (relation (72)).

Computation of long run price and income elasticities

In this section we summarize the major estimation results in the form of the implied long-run overall income and uncompensated price elasticities (see table 27). In order to obtain these elasticities we first check if the allocation module behaves well i.e. we check if it gives stable results when simulating for a long simulation period. For this purpose we simulate the allocation modules alone - i.e. not with the complete HERMES model - up to horizon 2100 keeping prices and total energy expenditure per industry constant. The adjustment process brings the systems to a steady state. Next we introduce (per industry) a 1 percent increase in total real energy expenditure or in individual prices of each energy category (period 2050). In period 2050 the deviations from the baseline can be interpreted as short-run income and uncompensated price elasticities, while the responses in period 2059 approximate the long-run elasticities. These 'equilibrium' elasticities obtained by simulating the allocation module alone differ from 'real world 'elasticities because of the absence of feedback mechanisms.

The results are presented per industry, for each energy product (coal, coke, oil, gas and electricity). Note that coke and coal are only consumed in a limited number of industries.

Table 27: Long-run Overall Own Uncompensated Price and Income Elasticities

Industries	Long-run Overall Own Uncompensated Price Elasticity	Long-run Overall Income Elasticity	Industries	Long-run Overall Own Uncompensated Price Elasticity	Long-run Overall Income Elasticity
Agriculture			**Health care**		
Coal/coke	(-)	(-)	Coal/coke	(-)	(-)
Oil	-0.49	0.63	Oil	-0.78	0.28
Gas	-0.99	1.00	Gas	-0.51	1.05
Electricity	-0.66	1.31	Electricity	-0.89	1.22
Intermediate goods			**Other market services to households and firms**		
Coal	-0.54	7.45	Coal/coke	-0.99	1.00
Coke	-0.50	1.31	Oil	-0.55	0.49
Oil	-0.35	1.63	Gas	-0.43	1.03
Gas	-0.82	1.18	Electricity	-0.82	1.54
Electricity	-0.58	0.61			
Equipment goods			**Rail and road transportation**		
Coal/coke	(-)	(-)	Coal/coke	(-)	(-)
Oil	-1.08	1.89	Oil	-0.94	1.00
Gas	-0.37	0.63	Gas	-0.24	0.44
Electricity	-0.04	0.15	Electricity	-0.16	1.00
Consumer goods			**Air, maritime and fluvial transportation**		
Coal/coke	(-)	(-)	Coal/coke	(-)	(-)
Oil	-0.75	0.43	Oil	-0.99	1.00
Gas	-1.31	1.88	Gas	-0.33	0.51
Electricity	-0.62	1.00	Electricity	-0.28	0.81
Construction			**Other transport services and communication**		
Coal/coke	(-)	(-)	Coal/coke	-0.99	1.00
Oil	-1.04	1.00	Oil	-0.67	0.37
Gas	-0.99	1.00	Gas	-0.25	0.66
Electricity	-1.03	1.00	Electricity	-1.19	1.63
Trade, lodging and catering services			**Non market services**		
Coal/coke	-0.62	0.75	Coal/coke	-0.98	0.41
Oil	-0.88	1.04	Oil	-0.54	0.61
Gas	0.05	0.31	Gas	-0.55	1.08
Electricity	-0.90	1.21	Electricity	-0.91	1.48
Services of credit and insurance institutions					
Coal/coke	-0.41	2.85			
Oil	-0.74	1.14			
Gas	-0.36	0.74			
Electricity	-0.85	1.01			

First note that all elasticities are obtained with the correct sign (positive for income elasticities; negative for own price elasticities). The implied long-run income elasticities can be relatively high (more than 1.0), depending on the industries and products. Many income elasticities are equal or close to 1.0. In some cases, income elasticities are relatively low (less than 0.5). The long-run uncompensated own price elasticities, which take into account the income effect of a price change, are mainly smaller than 1.0 (in absolute terms), but can be larger in some cases.

5.6. Environmental module

The environmental module is designed to model the interactions between economic activities, modelled in the other part of the model and the emissions of pollutants which come from these activities and the environmental policies which should be introduced.

5.6.1. Main technical relations

In this point, we specify the technical relations between the various forms of energy consumption and the emission of air pollutants.

a. Emissions from energy use

For the calculation of the energetic CO_2, CH_4 and N_2O emissions we multiply the energy consumption by industry or households and by energy product with the respective emission coefficient.

$$E_{x,j} = EF_{x,j} * ENER_{x,j} \tag{78}$$

With:

x = i or h (industry i or households);

$E_{x,j}$: emission per ton/year by industry (x= i) or households (x=h) and by energy product j;

$ENER_{x,j}$: energy consumption in Joule;

$EF_{x,j}$: emission factor in ton/Joule.

The emission factors for CO_2 emissions are product linked and similar for the different industries and households by energy product. The emission factors for the calculation of the energetic CO_2 emissions are conform with the Second Belgian National Communication.

The emission factors for CH_4 and N_2O are implicit emission factors and are calculated by dividing the emissions by industry or households by the energy consumption. The result for the latest observation year is used as implicit emission factor for the projection period. The emission factor is identical for all energy products by industry or households.

Technically there is no evolution in the emission factors. The emission factors remain constant during the projection period. However, more efficient energy consumption and the structural change in the composition of energy consumption (thanks to specific technological evolutions) are taken into account.

b. Other GHG emissions

For the calculation of the non-energetic CO_2, CH_4, N_2O, HFK, PFK, SF_6 emissions we multiply the emission explaining variable with the respective emission coefficient.

$$E_x = EF_x * EEV_x \tag{79}$$

With:

x = i or h;

E_x emission per ton/year by industry (x= i) or households (x=h);

EEV_x the emission explaining variable (ex. production for x=i consumption of households for x=h) in chained prices;

EF_x the emission factor in ton per unit of EEV.

The emission factors for the non-energetic CO_2, CH_4, N_2O, HFK, PFK, SF_6 emissions are implicit emission factors calculated by dividing the respective emissions from the inventories by the emission explaining variable from HERMES in chained prices. The result for the latest observation year is used as implicit emission factor for the projection period. We suppose constant policy and no specific technical evolution for these emissions. The emission factors are constant during the projection period.

c. Total GHG emissions

The following table gives an overview of the different GHG emitted in Belgium for the years 1995, 2000 and 2010.

As can be seen, total greenhouse gas emissions decreased from 1995 to 2010 by more than 10%. CO_2 emissions from energy combustion took the largest part in total greenhouse gas emissions and were close to 80% of total emissions. Non energetic CO_2 emissions come from industrial processes and decreased from 9.7 million ton (Mton) in 1995 to 8.2 Mton in 2010. CH_4 emissions decreased more rapidly over the period 1995-2010. The main source of CH_4 emissions is agriculture. N_2O emissions also decreased over the same period. N_2O emissions mainly come from industrial processes (production of nitric acid) and agriculture. The emission of fluorinated gases decreased drastically in 2000 thanks to important investments. Afterwards the emissions of fluorinated gases increased again.

Table 28: Greenhouse gas emissions in Belgium - 1995, 2000 and 2010
in Mton-CO_2 equivalents

	1995	2000	2010
CO_2 from energy combustion	114.7	115.4	106.7
CO_2 from industrial processes and waste	9.7	9.8	8.2
CH_4	9.3	8.3	6.5
N_2O	11.7	11.0	8.3
HFC, PFC, SF_6	5.0	1.4	2.1
Total	150.4	146.0	131.8

5.6.2. Modelling emission reduction scenarios

Different instruments can be considered in the framework of policies aimed at reducing GHG emissions. The use of market-based instruments for the control of carbon emissions is now widely accepted and, among them, environmental taxes (the so-called pigouvian tax) have been widely studied. Other market-based instruments concern for instance tradable permits.

Energy-related taxes and charges considered as environmental include:

– excise duties on energy and taxes based on the energy content of the energy source (as well as specified production taxes on hydro and nuclear power): the energy tax;

– taxes based on the carbon content of the fuel or CO_2 emitted in the fuel combustion process: CO_2 tax or carbon tax;

– taxes and charges based on sulphur content of the fuel or sulphur oxides emitted in the fuel combustion process: sulphur tax;

– taxes and charges based on nitrous oxide (NOX) emitted in fuel combustion process: nitrogen charge;

– excise duties on electricity production and taxes on electricity consumption: electricity tax.

An example of the use of such instruments can be found in the following recent studies published by the FPB. In the first study, the authors propose to increase the taxes on energy according to different modalities[37]: alignment of energy products prices (all taxes included) on the price observed in the three big neighbouring countries (Germany, France and the Netherlands); alignment of the taxation level up to the level observed in the three big neighbouring countries; alignment on the energy prices observed in the Nordic countries; alignment on the Danish energy prices. According to the method, energy prices can be increased from about 3% to more than 70% and in all cases, energy demand as well as GHG emissions can be significantly reduced. In the second study, a carbon tax attaining the equivalent of 24 euros per ton of CO_2 is imposed for all non-ETS sectors[38]. This kind of taxation, which increases average energy prices with 5 to 6%, also permits to reduce significantly energy demand and has a more important effect on CO_2 emissions because of substitution effects between fuels (in favour of less pollutant fuels) provoked by the targeted energy price taxation.

37 See Bassilière et al. (2009).

38 i.e. construction, transport, services, agriculture, residential and small industrial plants. The total CO_2 emissions of the non-ETS sector attained about 71.4 million tons in 2011. A tax of 24 euros/ton of CO_2 should give a gross receipt of EUR 1.70 billion, i.e. the equivalent of 0.5% of Belgian GDP. See Bassilière et al. (2010).

6. Public finances

The public finances section is an essential part of the HERMES model. The State intervenes at nearly all stages of the modelling process, such as price formation (VAT, excises, taxes on energy, subsidies,...), the computation of net disposable income of households (via direct taxes, social security contributions or benefits), which can affect the behaviour of the different categories of economic agents, or the formation of public investment policy, which directly and indirectly influences the different production branches.

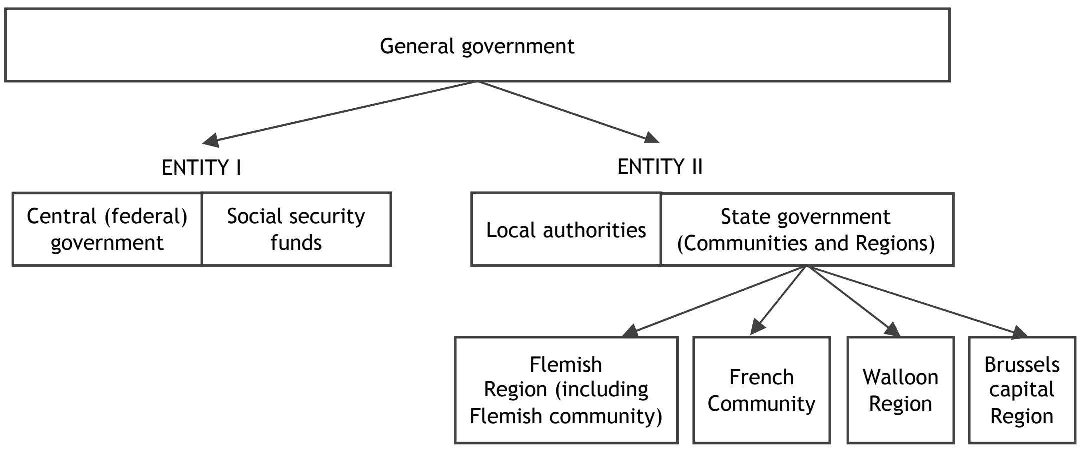
The public finance module of HERMES is characterized by a disaggregation of general government into four main subsectors (7 subsectors if we take into account the disaggregation of communities and regions):

- Central (or federal) government;
- State government (or communities and regions): disaggregated into Flemish Region (including Flemish Community), French Community, Walloon Region, Brussels-Capital Region;
- local authorities;
- Social security funds.

The four main subsectors are themselves grouped into two large entities (see box 3):

- Entity I: Central (federal) government and social security funds
- Entity II: Local authorities and State government (Communities and regions)

Box 3 Entities and sectors in the Belgian public administrations



For each subsector, the model computes a current transactions account and a capital account and determines respectively the operating surplus, the primary income, the disposable income, the current savings and the net financing or requirement capacity. The indebtedness is also computed (for the public administrations as a whole and for each subsector).

The current transactions of the general government can be divided into resources and expenditures. The capital transactions, on their turn, can also be divided into resources and uses. In the next point, we describe more precisely the different public resources and expenditures.

6.1. Resources

The resources of the four main subsectors of the general government are modelled, as far as possible, according to the macrosectoral approach. Indirect taxes and employers' social security contributions are based on the disaggregation of the national economy into the fifteen HERMES branches. Current taxes on income and wealth paid by households, corporate taxes and employees' social security contributions, however, are calculated only at the macroeconomic level.

6.1.1. Direct taxation

Direct taxation applies to taxes paid by households, taxes paid by non-profit institutions serving households (NPISH), corporate taxes and other direct taxes. In 2011, direct taxes attained a total amount of EUR 59 billion, i.e. 32% of total public receipts.

a. Taxes paid by households

Taxes paid by households (80% of all direct taxes - more than EUR 47 billion in 2011) mainly concern personal income tax (including surcharges) as well as advance tax payment on movable property, taxes paid by officials of international institutions, … Personal income tax is modelled as follows: calculation of taxable bases for different taxable persons (wage earners, self-employed, unemployed, …); reconstitution of the number of taxpayers per taxable base; and finally, based on the different aggregates calculated earlier, an elasticity equation is formulated which allows determining the dynamics of the income tax evolution. This elasticity is so that the elasticity of real income tax per head with respect to real tax base per head is higher than 1, the elasticity to prices is equal to 1 and the elasticity to the number of taxable persons is equal to 1[39].

b. Corporate taxes

Corporate taxes (about 20% of direct taxes) also distinguish taxes on movable assets and other taxes. Other corporate taxes depend on the evolution of the net operating surplus of firms and on the corresponding taxation rate (the nominal taxation attains about 34%).

[39] For a detailed presentation of the modelling of personal income tax, see Frogneux and Saintrain (2012).

6.1.2. Indirect taxation

Indirect taxes represented a total amount of EUR 47 billion in 2011. Two main categories of indirect taxes are distinguished:

– indirect taxes on products (EUR 40 billion in 2011): taxes on products are taxes due per unit of good or service produced or exchanged. They may correspond to a specified monetary amount due per unit of quantity of the good or service or be calculated *ad valorem* in the form of a fixed percentage of their unit price or of their value. Taxes on products mainly concern VAT, which in total amounted to EUR 26 billion in 2011 and excise duties (on energy products, tobacco, alcohol, ...), which in total amounted to nearly EUR 7 billion in 2011. Both VAT and excise duties are calculated in a relatively precise manner, at product or service level, when they are identified in the model. Taxes on products also include import duties and other taxes such as vehicle registration tax, tax on lotteries, gambling tax, premium tax, …

– indirect taxes on production (EUR 6.6 billion in 2011): the other taxes on production include all taxes paid by firms with regard to their production activities, irrespective of the quantity or value of goods and services produced or sold. Those taxes can be due on lands, fixed assets, employed labour force or certain activities or operations. Production taxes include, for instance, advance tax payment on buildings (EUR 4.6 billion in 2011) or the Eurovignette (EUR 135 million in 2011).

6.1.3. Social security contributions (SSC)[40]

Social security contributions (more than EUR 52 billion in 2011, i.e. 28.7% of total receipts), which form the main part of the revenues of the social security sector, are divided, in a first level, into employers' social security contributions, on the one hand, and contributions paid by employees, self-employed workers and non-employed persons, on the other hand. In a second level, a distinction has to be done between effective contributions and fictive contributions. Finally, the model allocates the contributions between the different beneficiary sectors (public administrations or firms).

Employers' social security contributions are computed at the level of the branch: the amount of each branch contribution is obtained by applying a SSC rate (which is the sum of the legal Social Security rate and the imputed SSC rate) to the endogenous wage bill per category of worker (low wage, high wage, special employment,...)[41]. Employees' contributions (as well as contributions paid by self-employed workers and non-employed persons) are not computed at the level of the branch, but directly for each category (employees from the private sector, employees from the public sector, self-employed workers and non-employed persons).

40 A more complete presentation concerning the computation of social security contributions can be found in Festjens et al. (2006).

41 See point 4.3.

6.1.4. Other receipts

Other public receipts concern:

– Property income: this kind of receipt is computed at the level of each sub-sector of the general administrations. According to the category of income, property revenues will depend on GDP or interest rates evolution; some categories are exogenous.

– Current and capital transfers are also computed for each sub-sector of the general administrations. Current and capital transfers are received from other internal sectors as well as from the rest of the world. These kinds of income can be exogenously fixed, or can be indexed on consumer prices, or depend on specific variables evolution (wages, social contributions,...).

6.2. Expenditures

The different types of government expenditures are all modelled on a macroeconomic level (except subsidies), according to a bottom-up approach (the expenditure of category x is the sum of category x expenditure computed for each sub-sector of the general government). A first distinction is made between primary expenditures and interest charges; primary expenditures are then broken down between current and capital expenditures. The main categories of expenditures are detailed in the next point.

6.2.1. Compensation of employees[42]

The compensation of employees constitutes one of the main categories of public current expenditures, with about 27% of all current expenditures in 2011. It includes:

– the gross wages and salaries;

– the actual employers' social contributions;

– the imputed employers' social contributions.

In HERMES, the public wage evolution depends on wage indexation (the index is computed elsewhere in the model), on the evolution of employment by broad category (general administration, national defence, education), on the wage drift and on wage revaluations. It should be noted that this calculation is made at the level of each public entity distinguished by the model (federal authorities, communities, regions, local authorities,...), as well as for public enterprises included in the institutional sector S.13.

6.2.2. Intermediate consumption and paid taxes

This second category represented almost 8% of all current public expenditures in 2011. Intermediate consumption is again calculated at the level of each public entity: for each of them, the intermediate

[42] For a more detailed presentation on this item, see Laloy (2007).

consumption is calculated on the basis of an exogenous growth (to which an indexation is added). The paid taxes are related to GDP.

6.2.3. Subsidies to enterprises

Not less than 6% of all current expenditures (in 2011) are devoted to this item. Most subsidies to enterprises are paid to the branch transport and communication (to the public enterprises: SNCB and BPOST). It should be noted that an increasing part (of the subsidies to enterprises) is granted in the form of wage subsidies. Subsidies to public enterprises are largely exogenous (in real terms), while wage subsidies are obtained by applying an exogenous rate to the endogenous wage bill per category of worker. In 2011, wage subsidies amounted to more than EUR 6 billion, i.e. 3.5% of public current expenditures or 5.5% of the private wage bill.

6.2.4. Social benefits

Social benefits reached in 2011 a total amount of EUR 93 billion, representing 54% of the total current expenditures of the general government. The national accounts distinguish two main categories of social benefits:

– Social benefits other than in kind: benefits which are paid in cash and that require no evidence of actual expenditure by the beneficiary. Cash expenditures may be periodic (unemployment allowances, pensions, family allowances,...) or unique (birth or death allowances,...);
– Social benefits in kind: those benefits are granted in the form of goods or services. The major part of social benefits in kind concerns all refunds in health care (about EUR 26 billion in 2011).

Generally, social benefits in cash are modelled taking into account the evolution of the number of beneficiaries (unemployed people, retired people,...), as well as the indexation coefficient usually used and any modification decided by the public authority.

As far as social benefits in kind are concerned, the real public health expenditures are explained on the basis of three structural determinants: income, age composition of the population and the medical technological progress. This last factor is measured by the number of patents granted for medical technology[43].

6.2.5. Transfers to households, NPISH, enterprises and the rest of the world

a. Transfers to households and NPISH

Transfers to households and NPISH attained a total of EUR 5.3 billion in 2011. The major part of transfers are paid by communities and regions. Real transfers are generally exogenous in projection, the nominal amounts being obtained by applying an indexation coefficient to the real amounts.

43 For a detailed presentation of the health care model used in HERMES, see Willemé (2011).

b. Transfers to enterprises

This item only concerned an amount of EUR 211 million in 2011. In projection, current transfers to enterprises are generally exogenously fixed (some of them being indexed).

c. Transfers to the rest of the world

Transfers to the rest of the world, including the « Gross National Income (GNI) contribution" paid to the European Union, mainly include expenditures for development cooperation and transfers to various European institutions. Transfers to the rest of the world are usually calculated exogenously in real terms and indexed to consumer prices, or, as it is the case for GNI contribution to the EU, they grow at the same rate as nominal GDP.

6.2.6. Capital expenditures

Capital expenditures attained EUR 11.5 billion in 2011. The gross capital formation represents the main component of this kind of expenditure (EUR 6.3 billion in 2011), followed by the capital transfers to enterprises (EUR 4.3 billion), capital transfers to the rest of the world (EUR 0.6 billion) and transfers to households and NPISH (EUR 0.4 billion).

- Gross capital formation: public investments are distributed among the different levels of power distinguished by the model. The largest portion of public investment is supported by local authorities and the various entities constituting the communities and regions. Investments are also made by the federal government, as well as by the social security administration. The different categories of investments which are distinguished are generally computed by choosing an exogenous evolution in real terms. This real evolution is then indexed to obtain a projection of nominal investments.
- Capital transfers to enterprises: are mainly paid by the federal government but also by regions and communities (and for a very limited amount by local authorities). Real transfers are generally exogenous in projection, an indexation being applied to compute the nominal amounts.
- Capital transfers to the rest of the world: consist, almost at 100%, of expenditures for development cooperation and of transfers to international organizations. These transfers are fixed in real terms and indexed.
- Capital transfers to households and NPISH: concern basically grants and subsidies granted to individuals to improve the housings' quality. These transfers are again projected in real terms and then indexed.

6.2.7. Interest charges

Interest charges represent the interests paid on borrowings made by the different levels of authority. Interest charges amounted to nearly EUR 12.8 billion in 2011, equivalent to 3.3% of GDP. Their calculation is function of the evolution of the interest rate and the debt, which depends itself on the evolution of public borrowings.

6.3. Net lending or net borrowing of the general government

The net lending or net borrowing of the general government is obtained by subtracting the total public expenditures from the total public receipts

$$FLG = REV\_PUB - DEP\_PUB \tag{80}$$

Where:

REV_PUB is the sum of public receipts (current and capital receipts);

DEP_PUB is the sum of public expenditures (current and capital expenditures).

Note that equation (80) can also be written as:

$$FLG = \sum_i FLG_i = \sum_i REV\_PUB_i - \sum_i DEP\_PUB_i \tag{81}$$

Where:

FLG_i is the net lending or net borrowing of each public entity (federal government, local authorities,...);

REV_PUB_i is the sum of receipts of entity i;

DEP_PUB_i is the sum of expenditures of entity i.

The model also computes the primary surplus or deficit, which is the difference between the total public receipts and the total public expenditures (without the interest charges)

$$PRIM = REV\_PUB - DEP\_PUB + INTDP \tag{82}$$

Where:

$INTDP$ represent the interest charges paid by the general government.

Finally, the government surplus/deficit used in the frame of the "excessive deficit procedure" (EDP) can be computed. It is obtained by adding net streams of interest payments resulting from swap arrangements and forward rate agreements to the net lending/net borrowing of the general government.

$$FLG\_EDP = FLG + SWAPS \tag{83}$$

7. Use of the model: medium-term outlooks and variants

This section is devoted to the presentation of the main uses of the model. Since its first version, HERMES has been used for two main purposes:

– Medium-term outlooks: yearly prepared and covering the next 5 to 7 years period;

– Policy and external shocks variants.

7.1. HERMES and the medium-term outlooks

The annual elaboration of a medium-term projection is one of the recurrent tasks of the FPB. Within the framework of the "European Semester", a first version of this multi-year projection is published during the month of March of each year, to provide a macroeconomic support for the development of the stability programme that Belgium must submit to the European Union. A final version, including the last short-term economic developments and the most recent information concerning the evolution of public finances, is presented in May to the Central Economic Council.

7.1.1. From the short to the medium term

Medium-term prospects that are carried out with HERMES always take as starting point the short-term forecast developed with other tools, more suitable for a short run analysis[44]. The medium-term projection may be considered, in this context, as the natural extension of the short-term exercise, notably intended to examine how imbalances identified in the short run (1 to 2 years) can be worked off (fully or partially) on a medium-term horizon. The model used for the medium term, however, is not confined to the variables used in the short term (especially in the case of the HERMES model), even if common assumptions (for example in the field of international environment) can be used in both approaches.

7.1.2. Estimating potential GDP and the output gap[45]

The concept of potential output can be defined as the level of output consistent with stable inflation. As such it constitutes an aggregate indicator of the supply-side capacity of an economy. Since potential output is not directly observable, it may be computed through a variety of methods[46]. Most international organisations rely on a method based upon a macroeconomic production function that allows potential output to be broken down into contributions from input factors and total factor productivity. In order to identify the underlying trends, this methodology uses statistical filters to smooth some of the input series. Due to the well-known end-point bias (difficulty of disentangling the cycle from the trend at the end of the sample), a widespread approach consists of applying a filter to an historical series that is supplemented by projected values. Following this approach, the FPB uses the methodology

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[44] On the short-term model used by the Federal Planning Bureau, see Hertveldt et al. (2003).

[45] More detailed information concerning this topic can be found in Lebrun (2009) and Lebrun (2011).

[46] See ECB (2000) for an overview of the methods available.

developed by the European Commission[47] but applies it to its own historical database extended using its medium-term projection. This approach ensures full compatibility between the potential GDP estimates and the medium-term macroeconomic scenario produced by the HERMES model.

This procedure allows computing the so-called output gap which is defined as the percentage deviation of the actual level of GDP from its potential level. Under "normal" business cycle conditions, we impose that the output gap is closed at the end of the five-year projection horizon. This practice means that no cyclical component remains and that potential and actual GDP coincide.

The EC method relies on a Cobb-Douglas production function with constant returns to scale:

$$YPOT = (LP * HT)^{\alpha}\, K^{1-\alpha}\, TFPT \tag{84}$$

where:

$YPOT$ represents potential GDP,

LP represents potential employment,

HT represents trend average hours worked,

K represents capital stock,

$TFPT$ represents trend total factor productivity,

α represents the average wage share.

This approach defines potential GDP as the output produced with a "normal" degree of utilisation of the production factors.

For capital, the full utilisation of the existing stock represents the maximum contribution to potential output and therefore no smoothing is required. Future values of the capital stock are produced by the HERMES model within the medium-term scenario.

For labour, the normal degree of utilisation is not as straightforward. Computing labour input defined in total hours worked implies different steps.

Firstly, trend average number of hours worked are obtained by applying the Hodrick-Prescott filter to the observed and projected values of the series. The cyclical movements are filtered out by the smoothing procedure but the trend remains downwards oriented over the projection period.

Secondly, potential employment is computed as:

$$LP = (PARTS * POPW) * (1 - NAIRU) \tag{85}$$

where:

$PARTS$ stands for trend participation rate,

$POPW$ for population of working age,

47 See D'Auria et al. (2010).

$NAIRU$ stands for structural unemployment rate.

The trend participation rate is identified using the Hodrick-Prescott filter. Multiplied by the actual and projected population of working age, it provides the trend labour force. The structural unemployment rate is defined as the rate consistent with stable, non-accelerating (wage) inflation, commonly called the NAIRU. In our methodology, it is computed by removing the cyclical component (the "unemployment gap") from the actual unemployment rate. The system is identified by adding a reduced-form Phillips curve linking the change in wage inflation to the unemployment gap and other explanatory variables. In the case of Belgium, the second difference in productivity and in the terms of trade as well as the lagged unemployment gap have been selected as additional variables. By imposing a functional form to the equations representing the trend and cyclical components, the whole system can be estimated using the Kalman filter technique[48]. Potential total hours worked are obtained by multiplying trend average hours worked with potential employment.

Total factor productivity can be computed using the Cobb-Douglas production function specification:

$$TFP = Y/((L * H)^{\alpha} K^{1-\alpha}) \tag{86}$$

The so-called Solow residuals capture, in addition to the measurement errors, the efficiency of both production factors (E_L and E_K) as well as their degree of utilization (CU_L and CU_K):

$$TFP = (E_L{}^{\alpha} E_K{}^{1-\alpha}) * \left(CU_L{}^{\alpha} CU_K{}^{1-\alpha}\right) \tag{87}$$

To obtain trend total factor productivity, the second component has to be filtered out. Planas et al. (2010) propose a method using a bivariate Kalman filter exploiting the link between the cyclical component of the Solow residuals and an indicator of capacity utilisation.

Potential GDP and the contributions from each production factor can so finally be computed using equation (84).

Note that potential growth and the output gap are computed *ex post*, as a check. It is generally admitted that the output gap should be close to or equal zero at the end of the (medium-term) projection period. Once the output gap is closed, effective growth is assumed to equal potential growth. The concept of potential output makes the link between medium and long-term macroeconomic projections.

This approach ensures full compatibility between the potential GDP estimates and the medium-term macroeconomic scenario produced by the HERMES model. It also allows to check that the basic property of our medium-term model is well respected, that is a gradual return to trend growth once the short-term shocks have been absorbed.

[48] See D'Auria et al., op.cit.

7.2. Variants simulated with the HERMES model

7.2.1. Introduction

HERMES is also used to compute the impact of policy measures and external shocks on the Belgian economy, and this relative to a baseline scenario, defined as the situation without measure nor shock. Examples of policy measures already tested in the past (see, for example, Bassilière et al. (2010)) include changes in employers' social security contributions (general or targeted at low-wage workers), in employees' social security contributions, in valued added tax, in excise duties, in personal income tax, in taxes on energy, in public investment, etc. ; examples of external shocks are a change in international oil price, in exchange rate, in Belgium's potential export markets, etc. Those variant analyses are regularly made, on own initiative or at the request of the government, in order to evaluate the effects of tax reforms projects, of tax shifting operations, of new labour policies, etc.

Hereafter we present the effects of three policy measures and of one external shock, computed with the new version of the model. The three policy measures investigated are an increase in public investment, a cut in employers' social security contributions targeted at low-wage workers and an increased VAT on private consumption; the external chock tested is an increase in the price of crude oil.

The aim of presenting here the results of some variants is not to have a comprehensive set of variants (as in Bassilière et al. (2005) for example); it's rather a way to illustrate the main properties and mechanisms of the model.

7.2.2. Description of the variants

All simulations cover a eight-year period (from year t to year t+7). The results tables hereafter report the most important differences between the variant and the baseline scenario for years t, t+3, t+5 and t+7.

An important consideration should be made about wage determination. As already explained above (see sub-section 4.3.5.a), there exists an "endogenous wages" version of HERMES and an "exogenous wages" version of HERMES. The aim of this sub-section being to describe empirically the main mechanisms of the model, the results presented hereafter only concern the variants computed with the "endogenous wages" version of HERMES. As already mentioned (cf. sub-section 4.3.5.a), the endogenous wages version links the gross wages at the industry level to the health index inflation, the industry's value added deflator, the unemployment rate, macroeconomic labour productivity and industry-specific labour productivity.

Another important assumption that has to be kept in mind when analyzing the results of the variants is that labour supply is supposed to be inelastic.

a. An increase in public investment

Public investment is increased each year by an amount equivalent to 0.5% of the corresponding baseline GDP. Therefore, the introduced amount gradually increases from EUR 1.96 billion in the first year up to EUR 2.49 billion in year t+7.

b. A reduction in the employers' social security contributions targeted at low-wage workers

The employers' social security contributions on low wages are reduced by an amount equivalent to 0.5% of the corresponding baseline GDP. Therefore, the (exogenous) implicit contribution rates on low wages of all industries (except agriculture) have all been decreased by a fixed number of percentage points, that is about 8 percentage points. This adjustment is sustained throughout the whole simulation period.

c. An increased VAT on private consumption

VAT on private consumption is permanently increased by an amount equivalent to 0.5% of the baseline GDP. This has been done by increasing all non-zero implicit VAT rates of the private consumption categories defined in HERMES by 1.4 percentage point. This adjustment is sustained throughout the whole simulation period.

d. An increased crude oil price

The world crude oil price is increased by 20% every year of the simulation period. Note that in addition to this 20% increase of the price of a barrel of oil, the simulated variant includes a modification of the other non-financial variables of the international environment (that is of prices of imported (non energy) goods and services and of Belgium's potential export markets). Those modifications were computed with the European model NEMESIS (see Baudewyns and Bossier, 2010).

7.2.3. Results of the variants

a. An increase in public investment

The main macroeconomic results of an increase by 0.5% of GDP in public investment are presented in table 29.

Table 29: Main macroeconomic results of the increase in public investment (endogenous wages)
(differences in % compared to baseline, unless otherwise mentioned)

	t	t+3	t+5	t+7
GDP	0.30	0.23	0.16	0.11
Private consumption	0.12	0.13	0.09	0.05
Investments	2.83	2.78	2.68	2.57
Exports	0	-0.01	-0.03	-0.05
Imports	0.34	0.40	0.39	0.38
Consumer prices	0.02	0.12	0.18	0.21
Unit labour cost	-0.06	0.32	0.45	0.49
Employment	0.15	0.19	0.14	0.08
Employment (in thousands)	6.84	8.82	6.59	4.04
Real disposable income of the households	0.13	0.13	0.10	0.06
Current external balance (% of GDP)	-0.29	-0.37	-0.40	-0.41
Public net borrowing (% of GDP)	-0.30	-0.32	-0.37	-0.43

The measure has a positive impact on economic activity: in the first year, GDP should increase by 0.30% and, in its wake, employment should gain 6 800 units. Subsequently the effect is fading. This is explained by the gradually reduced surplus (compared to the baseline) of purchasing power of the households - and, hence, of private consumption -, itself due to the progressively higher inflation. In the medium term, GDP should only be 0.11% above its level of the baseline scenario, which means that the multiplier should be below 1. "Only" 4 000 additional jobs should be created in t+7, compared to the baseline scenario.

The budgetary position of the general government is obviously deteriorating as a result of increased capital expenditures aimed at financing the additional public investments. The budgetary cost is however smaller *ex post* (0.43% of GDP in t+7) than *ex ante* (0.50% of GDP), thanks to increased economic activity and employment, which leads to additional public revenues (mainly indirect taxes and social security contributions). Next to the increase of capital expenditures, the level of current expenditures also increases mainly because of higher interest charges (since the increase of public investment is financed by debt).

b. A reduction in the employers'social security contributions targeted at low-wage workers

The main macroeconomic effects of a cut by 0.5% of GDP in social security contributions targeted at low-wage workers are summarized in table 30.

Table 30: Main macroeconomic results of the reduction in social security contributions targeted at low-wage workers (endogenous wages)
(differences in % compared to baseline, unless otherwise mentioned)

	t	t+3	t+5	t+7
GDP	0.11	0.27	0.29	0.27
Private consumption	0.20	0.43	0.45	0.43
Investments	0.17	0.39	0.41	0.33
Exports	0.06	0.14	0.15	0.13
Imports	0.08	0.17	0.17	0.15
Consumer prices	-0.17	-0.33	-0.34	-0.28
Unit labour cost	-1.30	-1.31	-1.10	-0.78
Employment	0.34	0.70	0.78	0.74
Employment (in thousands)	15.47	32.40	36.79	35.01
Real disposable income of the households	0.20	0.39	0.43	0.45
Current external balance (% of GDP)	-0.04	-0.06	-0.05	-0.05
Public net borrowing (% of GDP)	-0.34	-0.24	-0.23	-0.22

The reduction in social security contributions targeted at low-wage workers directly decreases firms' labour cost, which makes the production factor "labour" more attractive. Substitution effects are playing in favour of labour and, more particularly, in favour of low-wage workers. Many new jobs should so be created; however, it should take some time before the measure has its full impact ("only" 15 500 jobs created in year t; +35 000 in t+7). As the new jobs created are relatively poorly remunerated, the impact on private consumption should be relatively low (compared to a situation where the measure is not targeted at low-wage workers) (+0.43% in t+7).

Following the cut in social security contributions targeted at low-wage workers and, consequently, the decrease of the production costs of firms, the general price level decreases. Private consumption prices should be 0.28% lower than in the baseline scenario at the end of the projection period.

This measure should also result in a slight decrease of the current external balance (-0.05% of GDP in t+7). Lower production costs of firms imply lower export prices and, consequently, an increase of the volume of exports. At the same time, following the rise in domestic demand, the volume of imports should be higher than in the baseline scenario.

All in all, at the end of the projection period, real GDP should exceed its level of the baseline scenario by 0.27%.

In t+7, public net borrowing should decrease by 0.22% of GDP while the initial (*ex ante*) cost amounted 0.50% of GDP; it means that a bit more than half of the cost of the cut in social security contributions targeted at low-wage workers should be self-financed thanks to the impact of this measure on activity and employment. Total revenues should be much lower than in the baseline scenario (-1.68 billion in t+7), due to the initial shock of lowering social security contributions but also due to the negative price effect that shrinks the nominal tax bases. At the same time, however, new jobs are created, what increases the taxable income.

Total expenditures should also be lower than in the baseline scenario (-0.57 billion in t+7): the negative price effect associated with the decrease of public transfers following the drop in unemployment more than offsets the increase in interest charges, due to the deterioration of the primary balance.

c. An increased VAT on private consumption

The main macroeconomic effects of a VAT-increase of 0.5% of GDP are presented in table 31.

Table 31: Main macroeconomic results of the increased VAT on private consumption (endogenous wages)
(differences in % compared to baseline, unless otherwise mentioned)

	t	t+3	t+5	t+7
GDP	-0.05	-0.24	-0.24	-0.21
Private consumption	-0.04	-0.58	-0.65	-0.66
Investments	-0.14	-0.51	-0.56	-0.51
Exports	-0.03	-0.07	-0.05	-0.02
Imports	-0.02	-0.26	-0.29	-0.29
Consumer prices	0.78	1.04	1.02	0.96
Unit labour cost	0.56	0.44	0.18	-0.03
Employment	-0.12	-0.29	-0.28	-0.21
Employment (in thousands)	-5.57	-13.73	-13.07	-9.92
Real disposable income of the households	-0.45	-0.66	-0.70	-0.70
Current external balance (% of GDP)	0.00	0.16	0.21	0.24
Public net borrowing (% of GDP)	0.43	0.22	0.21	0.22

The increased VAT has a direct effect on all domestic prices. The prices of private consumption should therefore increase by 0.78% in comparison with the baseline scenario in year t and by 0.96% in year t+7.

Those higher costs result in a loss of competitiveness on foreign markets; exports should (slightly) decrease in comparison with the baseline scenario. They also result in a loss of profitability of firms, which should reduce their investments (-0.47% in t+7). Higher inflation also means a loss of purchasing power for households (-0.70% in t+7), which affects private consumption (-0.66% in t+7). As domestic demand decreases, imports also decrease (-0.29% in t+7). All in all, global activity (GDP) should be 0.05% lower (compared to the baseline scenario) in t, 0.21% in t+7. This loss of activity should be accompanied by a destruction of 5 600 jobs in t and 9 900 jobs in t+7. The current external balance (in % of GDP) goes up with 0.24% compared to the baseline scenario in the medium term, mainly thanks to the decrease of imports (terms of trade are unchanged).

The impact of a VAT-increase on public finances largely depends on price effects. With the implementation of this measure, indirect tax revenues increase *ex post* by about 0.46% of GDP (note that because of the deterrent effect of the tax increase on private consumption, the additional revenue from indirect taxes *ex post* is smaller than *ex ante*). Personal tax revenues as well as social security contributions, expressed as % of GDP, are expected to be lower than in the baseline scenario (negative impact of the job destruction on total wages). Public expenditures, expressed as % of GDP, should all be either equal or higher than in the baseline scenario (positive price effect (on wages, social benefits, …), increased unemployment allowances, …), except for the interest charges (higher primary balance). All in all, the budgetary position of the general government should be about 0.22% of GDP higher than in the baseline scenario in t+7.

d. An increased crude oil price

The main macroeconomic effects of a 20%-increase in the price of a barrel of crude oil are shown in table 32.

Table 32: Main macroeconomic results of the increased crude oil price (endogenous wages)
(differences in % compared to baseline, unless otherwise mentioned)

	t	t+3	t+5	t+7
GDP	-0.04	-0.35	-0.47	-0.59
Private consumption	-0.21	-1.08	-1.37	-1.59
Investments	-0.20	-1.04	-1.62	-2.07
Exports	-0.28	-0.94	-1.22	-1.40
Imports	-0.41	-1.47	-1.96	-2.26
Consumer prices	0.71	1.59	1.95	2.16
Unit labour cost	0.39	0.81	0.81	0.69
Employment	-0.09	-0.32	-0.35	-0.31
Employment (in thousands)	-3.89	-14.82	-16.68	-14.76
Real disposable income of the households	-0.71	-1.36	-1.59	-1.77
Current external balance (% of GDP)	-0.93	-1.02	-1.03	-1.04
Public net borrowing (% of GDP)	-0.21	-0.66	-0.85	-1.01

First note the effect of this external shock on Belgian inflation. The higher production (energy) costs are reflected in higher production prices and, hence, in higher selling prices, so that in year t, private consumption should be 0.71% more expensive than in the baseline scenario. In year t+7, the price level of private consumption should be 2.16% above the corresponding level in the baseline projection. Following this price increase, the purchasing power of households should fall and, in its wake, the volume of their consumption (-1.59% in t+7) and of their (housing) investments (-1.12%). Firms should reduce their investments too (-2.69%) as their profitability decreases following the increase of their production costs. For the same reason, Belgian exporters should become less competitive and exports should decrease (-1.40%). Finally, as domestic demand and exports are expected to decrease, imports should decrease too (-2.26%).

All in all, GDP in volume should shrink by 0.04% in year t. Afterwards the loss should progressively increase to -0.59% in year t+7. The reduced economic activity results in job losses (a loss of about 14 800 units by t+7). The deterioration of the current external balance is due to the deterioration of the terms of trade, in addition to the decrease of exports (and despite the sharp decline in imports).

Public finances are mainly affected by lower tax revenues (due to less economic activity), increased public consumption (due to higher inflation) and additional social benefits (due to higher inflation and increased unemployment). The higher oil price should result in an extra (relative to the baseline scenario) budgetary cost of EUR 0.8 billion in year t. In the medium term, the public balance is expected to be EUR 5.0 billion lower than in the corresponding year of the baseline projection.

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